This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

3Nb

L-1-

George Drummond Home
of Blairdrummond

QA
465
.G82
1787

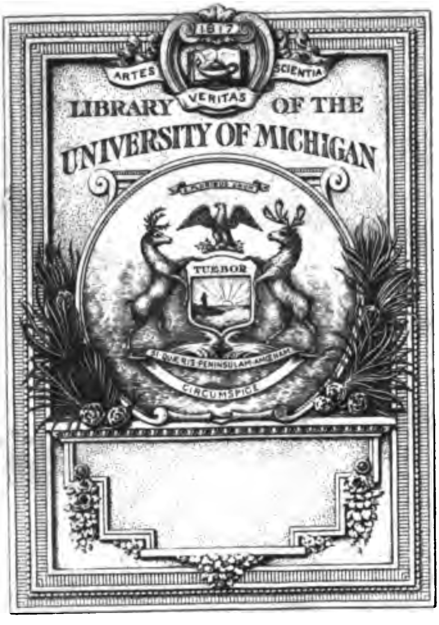
The Gift of

WILLIAM H. BUTTS, Ph.D.
A.B. 1878 A.M. 1879

Teacher of Mathematics
1898 to 1922

Assistant Dean, College of Engineering
1908 to 1922

Professor Emeritus
1922



H. Drummond Home. April 7th 17

A

TREATISE

OF

PRACTICAL GEOMETRY.

IN THREE PARTS.

By the late Dr DAVID GREGORY,

Some time Profeſſor of Mathematics in the Univerſity of EDINBURGH, and afterwards *Savilian* Profeſſor of Aſtronomy at OXFORD.

[Tranſlated from the Latin. With Additions.]

THE TENTH EDITION.

EDINBURGH:
PRINTED FOR JOHN BALFOUR.

M,DCC,LXXXVII.

W H Butts math fd
Bowes
2-4-31
33330

PREFACE.

THIS Treatiſe was compoſed in Latin, about ſixty years ago, by Dr DAVID GREGORY, then Profeſſor of Mathematics in the Univerſity of Edinburgh; where it has been conſtantly taught, ſince that time, immediately after Euclid's Elements, and the Plain Trigonometry, as proper for exerciſing the Students in the Application of Geometry to Practice. The Bookſeller having procured an Engliſh Tranſlation of it, which had been made by an ingenious Gentleman when a Student here, this Tranſlation has been reviſed, and ſeveral Additions have been made to the Treatiſe itſelf, in order to render it more uſeful at this time. The Reader will find theſe diſtinguiſhed from the Author's Text.

COL. M'LAURIN.

College of Edinburgh,
May 1. 1745.

A
TREATISE
OF
PRACTICAL GEOMETRY.

HAVING explained the firſt ſix books of Euclid, with the eleventh and twelfth, which may ſerve for geometrical elements, and having alſo taught the Plain Trigonometry, we are now to ſubjoin ſome corollaries which are eaſily deduced from them, that contain practical rules of great uſe in the affairs of life, concerning the menſuration of lines, angles, ſurfaces, and ſolids.

This Treatiſe of Practical Geometry is divided into three parts. In the firſt, we treat of the menſuration of lines and angles; to which we have ſubjoined ſurveying. In the ſecond, we treat of ſurfaces; not of ſuch as are plain only, but of ſome curve ſurfaces likewiſe; as of the ſurface of the cylinder, cone, and ſphere; and of thoſe parts of the ſphere which we have frequently occaſion to conſider. It is ſhewn how to expreſs the area of theſe in the ſuperficial meaſures that are now in uſe amongſt us. The third part treats of ſolid figures and their menſuration. After deducing the rules for finding the ſolid content of the parallelopipedon, priſm, pyramid, cylinder, cone, &c. from Euclid, we add, from Archimedes, the menſuration of the ſphere and ſpheroid, and of their ſegments, demonſtrated in an eaſy manner; from whence a method is derived for finding the contents of veſſels that are either full, or in part empty, in the wet as well as the dry meaſures, that are now in uſe amongſt us.

PART I.

A Line, or length, to be meaſured, whether it be diſtance, height, or depth, is meaſured by a line leſs than it. With us, the leaſt meaſure of length is an inch: Not that we meaſure no line leſs than it, but becauſe we do not uſe the name of any meaſure below that of an inch; expreſſing leſſer meaſures by the fractions of an inch: And in this Treatiſe we uſe decimal fractions as the eaſieſt. Twelve inches make a foot; three feet and an inch make the Scots ell; ſix ells make a fall; forty falls make a furlong; eight furlongs make a mile: So that the Scots mile is 1184 paces, accounting every pace to be five feet. Theſe things are according to the ſtatutes of Scotland; notwithſtanding which, the glaziers uſe a foot of only eight inches; and other artiſts, for the moſt part, uſe an Engliſh foot, on account of the ſeveral ſcales marked on the Engliſh foot-meaſure for their uſe. But the Engliſh foot is ſomewhat leſs than the Scots; ſo that 185 of theſe make 186 of thoſe.

Lines, to the extremities, and any intermediate point, of which you have eaſy acceſs, are meaſured by applying to them the common meaſure a number of times. But lines, to which you cannot have ſuch acceſs, are meaſured by methods taken from Geometry; the chief whereof we ſhall here endeavour to explain. The firſt is by the help of the geometrical ſquare.

" As for the Engliſh meaſures, the yard
" is three feet, or thirty-ſix inches. A pole
" is ſixteen feet and a half, or five yards
" and a half. The chain commonly called
" Gunter's chain, is four poles, or twenty-
" two yards, that is, ſixty-ſix feet. An
" Engliſh ſtatute mile is fourſcore chains,
" or 1760 yards, that is, 5280 feet.

“ The chain, (which is now much in uſe, becauſe it is very convenient for ſurveying), is divided into a hundred links, each of which is $7\frac{92}{100}$ of an inch: Whence it is eaſy to reduce any number of thoſe links to feet, or any number of feet to links.

“ A chain that may have the ſame advantages in ſurveying in Scotland, as Gunter's chain has in England, ought to be in length ſeventy-four feet, or twenty-four Scots ells, if no regard is had to the difference of the Scots and Engliſh foot above mentioned. But, if regard is had to that difference, the Scots chain ought to conſiſt of $74\frac{2}{5}$ Engliſh feet, or 74 feet 4 inches and $\frac{4}{5}$ of an inch. This chain being divided into an hundred links, each of thoſe links is 8 inches and $\frac{928}{1000}$ of an inch. In the following table, the moſt noted meaſures are expreſſed in Engliſh inches, and decimals of an inch.”

	Inch.	Dec.
The Engliſh foot is .	12	000
The Paris foot . .	12	788
The Rhinland foot, meaſured by Mr Picart . . .	12	362
The Scots foot . .	12	065
The Amſterdam foot, by Snellius and Picart . . .	11	172
The Dantzick foot, by Hevelius	11	297
The Daniſh foot, by Mr Picart	12	465
The Swediſh foot, by the ſame	11	692
The Bruſſels foot, by the ſame	10	828
The Lyons foot, by Mr Auzout	13	458
The Bononian foot, by Mr Caſſini	14	938
The Milan foot, by Mr Auzout	15	631
The Roman palm uſed by merchants, according to the ſame	9	791
The Roman palm uſed by architects . . .	8	779
The palm of Naples, according to Mr Auzout . .	10	314
The Engliſh yard . .	36	000
The Engliſh ell . .	45	000
The Scots ell . . .	37	200

	Inch	*Dec.*
The Paris aune uſed by mercers, according to Mr Picart	46	786
The Paris aune uſed by drapers, according to the ſame	46	680
The Lyons aune, by Mr Auzout	46	570
The Geneva aune	44	760
The Amſterdam ell	26	800
The Daniſh ell, by Mr Picart	24	930
The Swediſh ell	23	380
The Norway ell	24	510
The Brabant, or Antwerp ell	27	170
The Bruſſels ell	27	260
The Burges ell	27	550
The brace of Bononia, according to Auzout	25	200
The brace uſed by architects in Rome	30	730
The brace uſed in Rome by merchants	34	270
The Florence brace uſed by merchants, according to Picart	22	910
The Florence geographical brace	21	570
The vara of Seville	33	127

The

	Inch	Dec.
The vara of Madrid . .	39	166
The vara of Portugal .	44	031
The cavedo of Portugal .	27	354
The ancient Roman foot .	11	632
The Perſian ariſh, according to Mr Greaves . . .	38	364
The ſhorter pike of Conſtantinople, according to the ſame	25	576
Another pike of Conſtantinople, according to Meſſrs Mallet and De la Porte. . .	27	920

PROPOSITION I.

PROBLEM I.

To Deſcribe the Structure of the Geometrical Square.

THE Geometrical Square is made of any ſolid matter, as braſs or wood, or of any four plain rulers joined together at right angles, (as in fig. 1.); where A is the centre, from which hangs a thread with a ſmall weight at the end, ſo as to be directed always to the centre. Each of the ſides BE and DE is divided into an hundred equal parts, or (if the ſides be long enough to admit of it) into a thouſand parts; C and F are two ſights, fixed on the ſide AD. There is moreover an index GH, which, when there is occaſion, is joined to the centre A, in ſuch manner as that it can move round, and remain in any given ſituation. On this index are two ſights perpendicular to the right line going from the centre of the inſtrument: Theſe are K and L. The ſide DE of the inſtrument is called the upright ſide; E the reclining ſide.

PROP. II. FIG. 2.

To meaſure an Acceſſible Height, AB, *by the help of a Geometrical Square, its diſtance being known.*

LET BR be an horizontal plane, on which there ſtands perpendicularly any line AB: Let BD, the given diſtance of the obſervator from the height, be 96 feet; let the height of the obſervator's eye be ſuppoſed 6 feet; and let the inſtrument, held by a ſteady hand, or rather leaning on a ſupport, be directed towards the ſummit A, ſo that one eye (the other being ſhut) may ſee it clearly through the ſights; the perpendicular or plum-line mean while hanging free, and touching the ſurface of the inſtrument: Let now the perpendicular be ſuppoſed to cut off on the right ſide KN 80 equal parts. It is clear that LKN, ACK, are ſimilar triangles; for the angles LKN, ACK, are right angles, and therefore equal: Moreover,

over, LN and AC are parallel, as being both perpendicular to the horizon; conſequently, by Prop. 29. B. 1. of Euclid, the angles KLN, KAC, are equal; wherefore, by the ſecond corollary and of the 32. Prop. B 1. of Euclid, the angles LNK and AKC, are likewiſe equal: So that, in the triangles NKL, KAC, (by the 4. Prop. of the 6. B. of Euclid) as NK : KL : : KC (*i. e.* BD) : CA; that is, as 80 to 100, ſo is 96 feet to CA. Therefore, by the rule of three, CA will be found to be 120 feet; and CB, which is 6 feet, being added, the whole height is 126 feet.

But, if the diſtance of the obſervator from the height, as BE, be ſuch, that, when the inſtrument is directed, as formerly, toward the ſummit A, the perpendicular falls on the angle P, oppoſite to H, the centre of the inſtrument, and BE or GG be given of 120 feet; CA will alſo be 120 feet. For, in the triangles HGP, ACG, equiangular, as in the preceding caſe, as PG : GH : : GC : CA. But PG is equal to GH; therefore GC is likewiſe equal to CA: That is, CA will be 120 feet, and the whole height 126 feet, as before.

Let the diſtance BF be 300 feet, and the perpendicular or plum-line cut off 40 equal parts from the reclining ſide: Now, in this caſe, the angles QAC, QZI, are equal, by the 29. Prop. 1. B. of Euclid. And, by the ſame Prop. the angles QZI, ZIS are equal; therefore the angle ZIS is equal to the angle QAC. But the angles ZSI, QCA are equal, being right angles; therefore in the equiangular triangles ACQ, SZI, by the 4. Prop. of the 6. B. of Euclid, it will be, as ZS : SI :: CQ : CA; that is, as 100 to 40, ſo is 300 to CA. Wherefore, by the rule of three, CA will be found to be of 120 feet. And, by adding the height of the obſervator, the whole BA will be 126 feet. Note, That the height is greater than the diſtance, when the perpendicular cuts the right ſide, and leſs, if it cut the reclined ſide; and that the height and diſtance are equal, if the perpendicular fall on the oppoſite angle.

SCHOLIUM. Fig. 3.

If the height of a tower to be meaſured, as above, end in a point, as in Fig. 3. the diſtance of the obſervator oppoſite to it is not CD, but is to be accounted from the perpendicular to the point A; that is, to CD muſt be added the half of the thickneſs of the tower, viz. BD: Which muſt likewiſe be underſtood in the following propoſitions, when the caſe is ſimilar.

PROP. III.

Prob. Fig. 4.

From the height of a Tower AB *given, to find a diſtance on the horizontal plane* BC, *by the Geometrical Square.*

LET the inſtrument be ſo placed, as that the mark C in the oppoſite plane may be ſeen through the ſights; and let it be obſerved how many parts are cut off by the perpendicular. Now, by what hath been already demonſtrated, the triangles AEF, ABC are ſimilar; therefore, by 4th, 6. Eucl. it will be as EF to AE, ſo AB (compoſed of the height of the tower BG, and of the height of the centre of the inſtrument A, above the tower BG) to the diſtance BC. Wherefore, if, by the rule of three, you ſay, as EF to AE, ſo is AB to BC, it will be the diſtance ſought.

PROP. IV. FIG. 5.

To meaſure any diſtance at land or ſea by the Geometrical Square.

IN this operation, the index is to be applied to the inſtrument, as was ſhown in the deſcription; and, by the help of a ſupport, the inſtrument is to be placed horizontally at the point A; then let it be turned till the remote point F, whoſe diſtance is to be meaſured, be ſeen through the fixed ſights; and bring the index to be parallel with the other ſide of the inſtrument; obſerve by the ſights upon it any acceſſible mark B, at a ſenſible diſtance: Then carrying the inſtrument to the point B, let the immoveable ſights be directed to the firſt ſtation A, and the ſights of the index to the point F. If the index cut the right ſide of the ſquare, as in K, in the two triangles BRK, and BAF, which are equiangular, it will be (by 4th, 6. Eucl.) as BR to RK, ſo BA (the diſtance of the ſtations to be meaſured with a chain) to AF; and the diſtance AF ſought will be found by the rule of three. But, if the index cut the reclined ſide of the ſquare in any point L, where the diſtance of a more remote point is ſought; in the triangles BLS, BAG, the ſide LS ſhall be to SB, as BA to AG, the diſtance ſought; which accordingly will be found by the rule of three.

PROP.

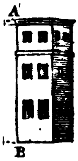
PROP. V.

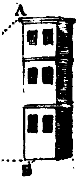
PROB. FIG. 6.

To meaſure an Acceſſible Height by means of a plain Mirror.

LET AB be the height to be meaſured; let the mirror be placed at C, in the horizontal plane BD, at a known diſtance BC; let the obſerver go back to D, till he ſee the image of the ſummit in the mirror, at a certain point of it, which he muſt diligently mark; and let DE be the height of the obſervator's eye. The triangles ABC and EDC are equiangular; for the angles at D and B are right angles; and ACB, ECD are equal, being the angles of incidence and reflection of the ray AC, as is demonſtrated in optics; wherefore the remaining angles at A and E are alſo equal: Therefore, by 4th, 6. Eucl. it will be, as CD to DE, ſo CB to BA; that this, as the diſtance

20





diſtance of the obſervator from the point of the mirror in the right line betwixt the obſervator and the height, is to the height of the obſervator's eye, ſo is the diſtance of the tower from that point of the mirror, to the height of the tower ſought; which therefore will be found by the rule of three.

Note 1. The obſervator will be more exact, if, at the point D, a ſtaff be placed in the ground perpendicularly, over the top of which the obſervator may ſee a point of the glaſs exactly in a line betwixt him and the tower.

Note 2. In place of a mirror, may be uſed the ſurface of water contained in a veſſel, which naturally becomes parallel to the horizon.

PROP.

PROP. VI. FIG. 7.

To meaſure an Acceſſible Height AB *by means of two ſtaffs.*

LET there be placed perpendicularly in the ground a longer ſtaff DE, likewiſe a ſhorter one FG, ſo as the obſervator may ſee A, the top of the height to be meaſured, over the ends D, F, of the two ſtaffs; let FH and DC, parallel to the horizon, meet DE and AB in H and C; then the triangles FHD, DCA, ſhall be equiangular; for the angles at C and H are right ones: Likewiſe the angle A is equal to the angle FDH, by 29th, 1. Eucl.; wherefore the remaining angles DFH, and ADC, are alſo equal: Wherefore, by 4th, 6. Eucl. as FH, the diſtance of the ſtaffs, to HD, the exceſs of the longer ſtaff above the ſhorter; ſo is DC, the diſtance of the longer ſtaff from the tower, to CA, the exceſs of the height of the tower above the longer ſtaff. And thence CA will be found by the rule of three.

To which, if the length DE be added, you will have the whole height of the tower BA. Q. E. F.

SCHOLIUM. Fig. 8.

Many other methods may be occaſionally contrived for meaſuring an acceſſible height. For example, from the given length of the ſhadow BD, I find out the height AB, thus: Let there be erected a ſtaff CE perpendicularly, producing the ſhadow EF: The triangles ABD, CEF, are equiangular; for the angles at B, and E, are right; and the angles ADB, and CFE, are equal, each being equal to the angles of the ſun's elevation above the horizon: Therefore, by 4th, 6. Eucl. as EF, the ſhadow of the ſtaff, to EC the ſtaff itſelf, ſo BD, the ſhadow of the tower, to BA, the height of the tower. Though the plane on which the ſhadow of the tower falls be not parallel to the horizon, if the ſtaff be erected in the ſame plane, the rule will be the ſame.

PROP. VII.

To meaſure an Inacceſſible Height by means of two ſtaffs.

HITHERTO we have ſuppoſed the height to be acceſſible, or that we can come at the lower end of it; now, if, becauſe of ſome impediment, we cannot get to a tower, or if the point whoſe height is to be found out be the ſummit of a hill, ſo that the perpendicular be hid within the hill; if, I ſay, for want of better inſtruments, ſuch an inacceſſible height is to be meaſured by means of two ſtaffs, let the firſt obſervation be made with the ſtaffs DE and FG, as in Prop. VI.; then the obſervator is to go off in a direct line from the height and firſt ſtation, till he come to the ſecond ſtation; where he is to place the longer ſtaff perpendicularly at RN, and the ſhorter ſtaff at KO, ſo that the ſummit A may be ſeen along their tops; that is, ſo that the points K, N, A may be in the ſame right line. Through the point N let there be drawn the right line NP parallel to FA: Wherefore, in the triangles KNP, KAF, the angles KNP, KAF are equal by the 29. 1. Eucl. alſo the angle AKF is common to both; conſequently the remaining angle KPN is equal to the remaining angle KFA. And therefore, by 4th, 6. Eucl. PN : FA : : KP : KF. But the triangles PNL, FAS are ſimilar; therefore, by 4th, 6. Eucl. PN : FA : : NL : SA. Therefore, by the 11th, 5. Eucl. KP : KF :: NL : SA. Thence, alternately, it will be, as KP (the exceſs of the greater diſtance of the ſhort ſtaff from the long one above its leſſer diſtance from it) to NL, the exceſs of the longer ſtaff above the ſhorter; ſo KF, the diſtance of the two ſtations of the ſhorter ſtaff, to SA, the exceſs of the height ſought above the height of the ſhorter ſtaff. Wherefore SA will be found by the rule of three. To which let the height of the ſhorter ſtaff be added, and the ſum will give the whole inacceſſible height BA. Q. E. F.

Note 1. In the ſame manner may an inacceſſible height be found by a geometrical ſquare, or by a plain ſpeculum. But we ſhall leave the rules to be found out by the ſtudent, for his own exerciſe.

Note 2. That by the height of the ſtaff we underſtand its height above the ground in which it is fixed.

Note 3. Hence depends the method of uſing other inſtruments invented by geometricians; for example, of the geometrical croſs: And, if all things be juſtly weighed, a like rule will ſerve for it as here. But we incline to touch only upon what is moſt material.

PROP. VIII. FIG. 9.

To meaſure the diſtance AB, *to one of whoſe extremities we have acceſs, by the help of four ſtaffs.*

LET there be a ſtaff fixed at the point A; then going back at ſome ſenſible diſtance in the ſame right line, let another be fixed in C, ſo as that both the points A and B be covered and hid by the ſtaff C; likewiſe going off in a perpendicular from the right line CB, at the point A, (the method of doing which ſhall be ſhown in the following *ſcholium*), let there be placed another ſtaff at H; and in the right line CKG, (perpendicular to the ſame CB, at the point B), and at the point of it K, ſuch that the points K, H, and B, may be in the ſame right line, let there be fixed a fourth ſtaff. Let there be drawn, or let there be ſuppoſed to be drawn, a right line HG parallel to CA. The triangles KGH, HAB, will be equiangular;

angular; for the angles HAB, KGH are right angles. Alſo, by 29th, 1. Eucl. the angles ABH, KHG are equal; wherefore, by the 4th, 6. Eucl. as KG (the exceſs of CK above AH) to GH, or to CA, the diſtance betwixt the firſt and ſecond ſtaff; ſo is AH, the diſtance betwixt the firſt and third ſtaff, to AB the diſtance ſought.

SCHOLIUM. Fig. 10.

To draw on a plane a right line AE perpendicular to CH, from a given point A; take the right lines AB, AD, on each ſide equal; and in the points B and D, let there be fixed ſtakes, to which let there be tied two equal ropes BE, DE, or one having a mark in the middle, and holding in your hand their extremities joined, (or the mark in the middle, if it be but one), draw out the ropes on the ground; and then, where the two ropes meet, or at the mark, when by it the rope is fully ſtretched, let there be placed a third ſtake at E; the right line AE will be perpendicular to CH in the point A, by 11th, 1. Eucl. In a manner not unlike to this, may any problems, that are reſolved by the ſquare and compaſſes, be done by ropes and a cord turned round as a radius.

PROP. IX. FIG. 11.

To meaſure the diſtance AB, *one of whoſe extremities is acceſſible.*

FROM the point A, let the right line AC, of a known length, be made perpendicular to AB, (by the preceding *ſcholium*): Likewiſe draw the right line CD perpendicular to CB, meeting the right line AB in D: Then, by the 8th, 6. Eucl. as DA : AC :: AC : AB. Wherefore, when DA and AC are given, AB will be found by the rule of three. Q. E. F.

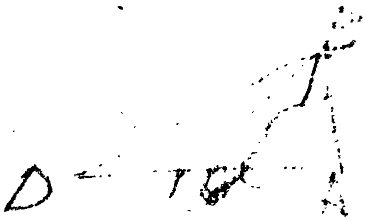
SCHOLIUM.

All the preceding operations depend on the equality of ſome angles of triangles, and on the ſimilarity of the triangles ariſing from that equality. And on the ſame principles depend innumerable other operations, which a geometrician will find out of himſelf, as is very obvious. However, ſome of theſe operations require ſuch exactneſs in the work, and without it are ſo liable to errors, that, *caeteris paribus*, the following operations, which are performed by a trigonometrical calculation, are to be preferred; yet could we not omit thoſe above, being moſt eaſy in practice, and moſt clear and evident to thoſe who have only the firſt elements of geometry. But, if you are provided with inſtruments, the following operations are more to be relied upon. We do not inſiſt on the eaſieſt caſes to thoſe who are ſkilled in plain trigonometry, which is indeed neceſſary to any one who would apply himſelf to practice. It would be eaſy to the reader to find examples; and we have ſhown, in plain trigonometry, how to find the angle or ſide of any plain triangle that is required, from the angles or ſides that may be given.

PROP.

ABC

There is given A 20°26′



also ∠ ADB = 10°,14′,
base 2130 [illegible] it is required
to find AB.

[illegible]
∠ GCA = 2[illegible]

also [illegible]
required to find AB.

UNIV. OF MICH.

PROP. X. FIG. 12.

To deſcribe the Conſtruction and Uſe of the Geometrical Quadrant.

THE Geometrical Quadrant is the fourth part of a circle divided into ninety degrees, to which two ſights are adapted, with a perpendicular or plumb line hanging from the centre. The general uſe of it is for inveſtigating angles in a vertical plane, comprehended under right lines going from the centre of the inſtrument, one of which is horizontal, and the other is directed to ſome viſible point. This inſtrument is made of any ſolid matter, as wood, copper, &c.

PROP. XI. FIG. 13.

To deſcribe the Make and Uſe of the Graphometer.

THE Graphometer is a ſemicircle made of any hard matter, of wood, for example, or braſs, divided into 180 degrees; ſo fixed on a *fulcrum*, by means of a braſs ball and ſocket, that it eaſily turns about, and retains any ſituation; two ſights are fixed on its diameter. At the centre there is commonly a magnetical needle in a box. There is likewiſe a moveable ruler, which turns round the centre, and retains any ſituation given in it. The uſe of it is to obſerve any angle, whoſe vertex is at the centre of the inſtrument in any plane, (though it is moſt commonly horizontal, or nearly ſo), and to find how many degrees it contains.

PROP. XII.

FIG. 14. and 15.

To deſcribe the manner in which Angles are meaſured by a Quadrant or Graphometer.

LET there be an angle in a vertical plane, comprehended between a line parallel to the horizon HK, and the right line RA, coming from any remarkable point of a tower or hill, or from the ſun, moon, or a ſtar. Suppoſe that this angle RAH is to be meaſured by the quadrant: Let the inſtrument be placed in the vertical plane, ſo as that the centre A may be in the angular point: And let the ſights be directed towards the object at R, (by the help of the ray coming from it, if it be the ſun or moon, or by the help of the viſual ray, if it is any thing elſe), the degrees and minutes in the arch BC, cut off by the perpendicular, will meaſure the angle RAH required. For, from the make of the quadrant, BAD is a right angle; therefore BAR is likewiſe right, being equal to it. But, becauſe HK is horizontal, and AC perpendicular, HAC will be a right angle; and therefore equal alſo to BAR. From thoſe angles ſubtract the part HAB that is common to both; and there will remain the angle BAC equal to the angle RAH. But the arch BC is the meaſure of the angle BAC; conſequently it is likewiſe the meaſure of the angle RAH.

Note, That the remaining arch on the quadrant DC is the meaſure of the angle RAZ, comprehended between the foreſaid right line RA and AZ, which points to the zenith.

Let it now be required to meaſure the angle ACB (Fig. 15.) in any plane, comprehended between the right lines AC and BC, drawn from two points A and B, to the place of ſtation C. Let the graphometer be placed at C, ſupported by its *fulcrum* (as was ſhown above); and let the immoveable ſights on the ſide of the inſtrument DE be directed towards the point A; and likewiſe (while the inſtrument remains immoveable) let the ſights of the ruler FG (which is moveable about the center C) be directed to the point B. It is evident that the moveable ruler cuts off an arch DH, which is the meaſure of the angle ACB ſought. Moreover, by the ſame method, the inclination of CE, or of FG, may be obſerved with the meridian line, which is pointed out by the magnetic needle incloſed in the box, and is moveable about the center of the inſtrument, and the meaſure of this inclination or angle found in degrees.

PROP. XIII. Fig. 16.

To meaſure an Acceſſible Height by the Geometrical Quadrant.

BY the 12th Prop. of this part, let the angle C be found by means of the quadrant. Then in the triangle ABC, right-angled at B, (BC being ſuppoſed the horizontal diſtance of the obſervator from the tower), having the angle at C, and the ſide BC, the required height BA will be found by the 3d caſe of plain trigonometry.

PROP. XIV. FIG. 17.

To meaſure an Inacceſſible Height by the Geometrical Quadrant.

LET the angle ACB be obſerved with the quadrant (by the 12th Prop. of this part;) then let the obſerver go from C to the ſecond ſtation D, in the right line BCD, (providing BCD be a horizontal plane;) and, after meaſuring this diſtance CD, take the angle ADC likewiſe with the quadrant. Then, in the triangle ACD, there is given the angle ADC, with the angle ACD; becauſe ACB was given before: Therefore (by 32. 1. Eucl.) the remaining angle CAD is given likewiſe. But the ſide CD is likewiſe given, being the diſtance of the ſtation C and D; therefore (by the firſt caſe of oblique angled triangles in Trigonometry) the ſide AC will be found. Wherefore, in the right-angled triangle ABC, all the angles and the hypotheneuſe AC are given; conſequently, by the 4th caſe of Trigonometry, the height ſought AB will be found; as alſo (if you pleaſe) the diſtance of the ſtation C from AB, the perpendicular within the hill or inacceſſible height.

PROP. XV. FIG. 18.

From the top of a given height, to meaſure the diſtance BC.

LET the angle BAC be obſerved by the 12th of this part; wherefore, in the triangle ABC, right-angled at B, there is given, by obſervation, the angle at A; whence (by the 32d, 1. Eucl.) there will alſo be given the angle BCA. Moreover, the ſide AB (being the height of the tower) is ſuppoſed to be given. Wherefore, by the 3d caſe of Trigonometry, BC, the diſtance ſought, will be found.

PROP. XVI. FIG. 19.

To meaſure the diſtance of two places A *and* B, *of which one is acceſſible, by the Graphometer.*

LET there be erected at two points A and C, ſufficiently diſtant, two viſible ſigns; then (by the 12th of this) let the two angles BAC, BCA, be taken by the Graphometer. Let the diſtance of the ſtations A and C be meaſured with a chain. Then the third angle B being known, and the ſide AC being likewiſe known; therefore, by the firſt caſe of Trigonometry, the diſtance required, AB, will be found.

PROP. XVII. Fig. 20.

To meaſure by the Graphometer, the diſtance of two places, neither of which is acceſſible.

LET two ſtations, C and D, be choſen, from each of which the places may be ſeen whoſe diſtance is ſought: Let the angles ACD, ACB, BCD, and likewiſe the angles BDC, BDA, CDA, be meaſured by the Graphometer; let the diſtance of the ſtations C and D be meaſured by a chain, or (if it be neceſſary) by the preceding practice. Now, in the triangle ACD, there are given two angles ACD and ADC; therefore the third, CAD, is likewiſe given. Moreover, the ſide CD is given; therefore, by the firſt caſe of Trigonometry, the ſide AD will be found. After the ſame manner, in the triangle BCD, from all the angles and one ſide CD given, the ſide BD is found. Wherefore, in the triangle ADB, from the given ſides DA and DB, and the angle ADB contained by them, the ſide AB (the diſtance ſought) is found by the fourth caſe of Trigonometry of oblique-angled triangles.

Let it be noted, that it is not neceſſary that the points A, B, C, and D, be in one plane; and that any triangle is in one plane, by 2d Prop. 11th of Eucl.

PROP. XVIII. Fig. 21.

It is required by the Graphometer and Quadrant, to meaſure an Acceſſible Height AB, *placed ſo on a ſteep, that one can neither go near it, in an horizontal plane, nor recede from it, as we ſuppoſe in the ſolution of the 14th* Prop.

LET there be choſen any ſituation, as C, and another, D; where let ſome mark be erected: Let the angles ACD and ADC be found by the Graphometer; then the third angle DAC will be known. Let the ſide CD, the diſtance of the ſtations, be meaſured with a chain, and thence (by Trigon.) the ſide AC will be found. Again, in the triangle ACB, right-angled at B, having found by the Quadrant the angle ACB, the other angle CAB is known likewiſe: But the ſide AC in the triangle ADC is already known; therefore the height required, AB, will be found by the 4th caſe of right-angled triangles. If the height of the tower is wanted, the angle BCF will be found by the Quadrant; which being taken from the angle ACB, already known, the angle ACF will remain: But the angle FAC was known before; therefore the remaining angle AFC will be known. But the ſide AC was alſo known before; therefore, in the triangle AFC, all the angles, and one of the ſides, AC, being known, AF, the height of the tower above the hill, will be found by Trigonometry.

SCHOLIUM.

It were eaſy to add many other methods of meaſuring heights and diſtances; but, if what is above be underſtood, it will be eaſy (eſpecially for one that is verſed in the elements) to contrive methods for this purpoſe, according to the occaſion: So that there is no need of adding any more of this ſort. We ſhall ſubjoin here a method by which the diameter of the earth may be found out.

PROP. XIX. FIG. 22.

To find the Diameter of the Earth from one obſervation.

LET there be choſen a high hill AB, near the ſea-ſhore, and let the obſervator on the top of it, with an exact Quadrant, divided into minutes and ſeconds by tranſverſe diviſions, and fitted with a teleſcope,

ſcope, in place of the common ſights, meaſure the angle ABE contained under the right line AB, which goes to the centre, and the right line BE, drawn to the ſea, a tangent to the globe at E; let there be drawn from A, perpendicular to BD, the line AF meeting BE in F. Now, in the right-angled triangle, BAF, all the angles are given, alſo the ſide AB, the height of the hill; which is to be found by ſome of the foregoing methods, as exactly as poſſible; and, by Trigonometry, the ſides BF and AF are found. But, by Corol. 36. 3. Eucl. AF is equal to FE; therefore BE will be known. Moreover, by 36th, 3. Eucl. the rectangle under BA and BD is equal to the ſquare of BE. And thence, by 17th, 6. Eucl. as AB : BE :: BE : BD. Therefore, ſince AB and BE are already given, BD will be found by 11th, 6. Eucl. or by the rule of three; and, ſubſtracting BA, there will remain AD, the diameter of the earth ſought.

SCHO-

SCHOLIUM.

Many other methods might be propoſed for meaſuring the diameter of the earth. The moſt exact, in my opinion, is that propoſed by Mr *Picart*, of the Academy of Sciences at *Paris*. But, ſince it does not belong to this place, we refer you to the Philoſophical Tranſactions, where you will find it deſcribed.

" According to Mr *Picart*, a degree of " the meridian at the latitude of 49° 21′, " was 57,060 *French Toiſes*, each of which " contains ſix feet of the ſame meaſure; " from which it follows, that, if the earth " be an exact ſphere, the circumference of " a great circle of it will be 123,249,600 " *Paris* feet, and the ſemidiameter of the " earth, 19,615,800 feet. But the *French* " Mathematicians, who of late have exa- " mined Mr *Picart's* operations, aſſure us, " That the degree in that latitude is 57,183 " *Toiſes*. They meaſured a degree in *Lap-* " *land*, in the latitude of 66° 20′, and found "it of 57,438 *Toiſes*. By comparing theſe "degrees, as well as by the obſervations on "pendulums, and the theory of gravity, it "appears that the earth is an oblate ſphe- "roid; and (ſuppoſing thoſe degrees to be "accurately meaſured) the axis or diame- "ter that paſſes through the poles will be "to the diameter of the equator as 177 to "178, or the earth will be 22 miles higher "at the equator than at the poles. A de- "gree has likewiſe been meaſured at the "equator, and found to be conſiderably "leſs than at the latitude of *Paris*; which "confirms the oblate figure of the earth. "But an account of this laſt menſuration "has not been publiſhed as yet. If the "earth was of an uniform denſity from the "ſurface to the centre, then, according to "the theory of gravity, the meridian would "be an exact ellipſis, and the axis would "be to the diameter of the equator as 230 "to 231; and the difference of the ſemidi- "ameter of the equator and ſemiaxis about "17 miles."

In what follows, a figure is often to be laid down on paper, like to another figure given; and becauſe this likeneſs conſiſts in the equality of their angles, and in the ſides having the ſame proportion to each other, (by the definitions of the 6th of Eucl.) we are now to ſhew what methods practical Geometricians uſe for making on paper an angle equal to a given angle, and how they conſtitute the ſides in the ſame proportion. For this purpoſe they make uſe of a Protractor, (or, when it is wanting, a Line of chords), and of a Line of equal parts.

PROP. XX.

FIG. 23. 24. 25. 26. and 27.

To deſcribe the Conſtruction and Uſe of the Protractor, of the Line of Chords, and of the Line of Equal Parts.

THE Protractor is a ſmall ſemicircle of braſs, or ſuch ſolid matter. The ſemicircumference is divided into 180 degrees.

The uſe of it is, to draw angles on any plane, as on paper, or to examine the extent of angles already laid down. For this laſt purpoſe, let the ſmall point in the centre of the protractor be placed above the angular point, and let the ſide AB coincide with one of the ſides that contain the angle propoſed; the number of degrees cut off by the other ſide, computing on the Protractor from B, will ſhow the quantity of the angle that is to be meaſured.

But, if an angle is to be made of a given quantity on a given line, and at a given point of that line, let AB coincide with the given line, and let the centre A of the inſtrument be applied to that point. Then let there be a mark made at the given number of degrees; and a right line drawn from that mark to the given point, will conſtitute an angle with the given right line, of the quantity required; as is manifeſt.

This is the moſt natural and eaſy method, either for examining the extent of an angle on paper, or for deſcribing on paper an angle of a given quantity.

But, when there is ſcarcity of inſtruments, or becauſe a line of chords is more eaſily carried about, (being deſcribed on a ruler on which there are many other lines beſides), practical Geometricians frequently make uſe of it. It is made thus: Let the quadrant of a circle be divided into 90 degrees, (as in Fig. 24.) The line AB is the chord of 90 degrees; the chord of every arch of the quadrant is transferred to this line AB, which is always marked with the number of degrees in the correſponding arch.

Note, That the chord of 60 degrees is equal to the radius, by Corol. 15th 4. Eucl. If now a given angle EDF is to be meaſured by the line of chords, from the centre D, with the diſtance DG, (the chord of 60 degrees), deſcribe the arch GF; and let the points G and F be marked where this arch interſects the ſides of the angle. Then, if the diſtance GF, applied on the line of chords from A to B, gives (for example) 25 degrees, this ſhall be the meaſure of the angle propoſed.

When an obtuſe angle is to be meaſured with this line, let its complement to a ſemicircle be meaſured, and thence it will be known. It were eaſy to transfer to the diameter of a circle the chords of all arches to the extent of a ſemicircle; but ſuch are rarely found marked upon rules.

But now, if an angle of a given quantity, ſuppoſe of 50 degrees, is to be made at a given point M of the right line KL (Fig. 26.) from the centre M, and the diſtance MN, equal to the chord of 60 degrees, deſcribe the arch QN. Take off an arch NR, whoſe chord is equal to that of 50 degrees on the line of chords; join the points M and R; and it is plain that MR ſhall contain an angle of 50 degrees with the line KL propoſed.

But ſometimes we cannot produce the ſides, till they be of the length of a chord of 60 degrees on our ſcale; in which caſe it is fit to work by a circle of proportions, (that is a Sector), by which an arch may be made of a given number of degrees to any radius.

The quantities of angles are likewiſe determined by other lines uſually marked upon rules, as the lines of ſines, tangents, and ſecants; but, as theſe methods are not ſo eaſy or ſo proper in this place, we omit them.

To delineate figures ſimilar or like to others given, beſides the equality of the angles, the ſame proportion is to be preſerved among the ſides of the figure that is to be delineated, as is among the ſides of the figures given. For which purpoſe, on the rules uſed by artiſts, there is a line divided into equal parts, more or leſs in number, and greater or leſſer in quantity, according to the pleaſure of the maker.

A foot is divided into inches; and an inch, by means of tranſverſe lines, into 100 equal parts; ſo that, with this ſcale, any number of inches, below twelve, with any part of an inch, can be taken by the compaſſes, providing ſuch part be greater than the one hundredth part of an inch. And this exactneſs is very neceſſary in delineating the plans of houſes, and in other caſes.

PROP. XXI. FIG. 28.

To lay down on paper, by the Protractor or line of chords, and line of equal parts, a right-lined figure like to one given, providing the angles and ſides of the figure given be known by obſervation or menſuration.

FOR example, ſuppoſe that it is known that, in a quadrangular figure, one ſide is of 235 feet, that the angle contained by it and the ſecond ſide is of 84°, the ſecond ſide of 288 feet, the angle contained by it and the third ſide of 72°, and that the third ſide is 294 feet. Theſe things being given, a figure is to be drawn on paper like to this quadrangular figure. On your paper, at a proper point A, let a right line be drawn, upon which take 235 equal parts, as AB. The part repreſenting a foot is taken greater or leſſer, according as you would have your figure greater or leſs. In the adjoining figure, the 100th part of an inch is taken for a foot. And, accordingly, an inch divided into 100 parts, and annexed to the figure, is called a ſcale of 100 feet. Let there be made at the point B (by the preceding Prop.) an angle ABC of 85°, and let BC be taken of 288 parts like to the former. Then let the angle BCD be made of 72°, and the ſide CD of 294 equal parts. Then let the ſide AD be drawn; and it will complete the figure like to the figure given. The meaſures of the angle A and D can be known by the protractor or line of chords, and the ſide AD by the line of equal parts; which will exactly anſwer to the correſponding angles, and to the ſide of the primary figure.

FIG. 22.

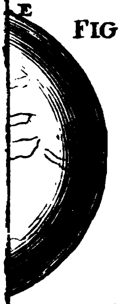
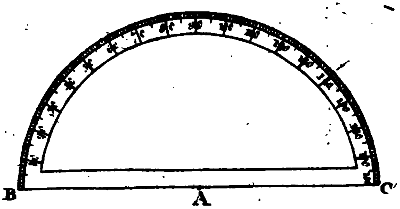


FIG. 23.



24.

FIG. 26.

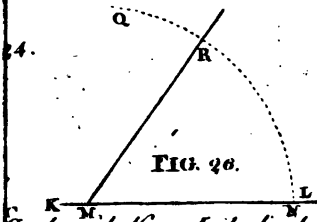
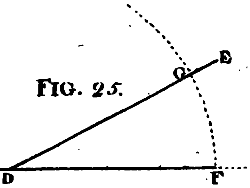
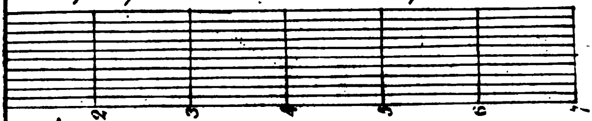


FIG. 25.



Scale of half an Inch divided into 100 parts



28.

FIG. 29.

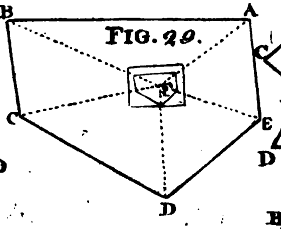


FIG. 31.

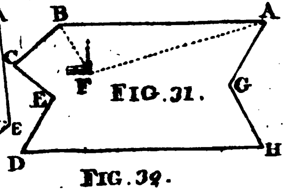


FIG. 32.

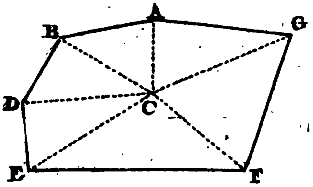
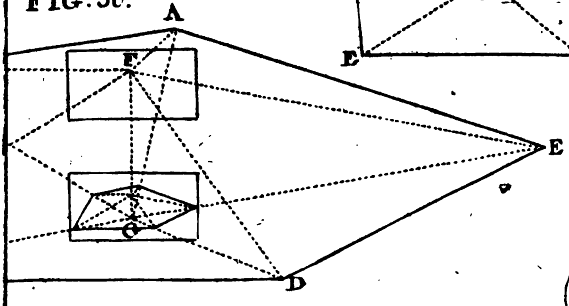


FIG. 30.



After the very ſame manner, from the ſides and angles given, which bound any right-lined figure, a figure like to it may be drawn, and the reſt of its ſides and angles be known.

COROLLARY.

Hence any trigonometrical problem in right-lined triangles, may be reſolved by delineating the triangle from what is given concerning it, as in this propoſition. The unknown ſides are examined by a line of equal parts, and the angles by a protractor or line of chords.

PROP. XXII. PROB.

The diameter of a circle being given, to find its circumference nearly.

THE periphery of any polygon inſcribed in the circle is leſs than the circumference, and the periphery of any polygon deſcribed about a circle is greater than the circumference. When Archimedes firſt diſcovered that the diameter was in proportion to the circumference, as 7 to 22 nearly, which ſerves for common uſe. But the moderns have computed the proportion of the diameter to the circumference to greater exactneſs. Suppoſing the diameter 100, the periphery will be more than 314, but leſs than 315 *. But Ludolphus Van Cuelen exceeded the labours of all; for, by immenſe ſtudy, he found, that, ſuppoſing the diameter

100,000,000,000,000,000,000,000,000,000,000,

the periphery will be leſs than

314,159,265,358,979,323,846,264,338,327,951,

but greater than

314,159,265,358,979,323,846,264,338,327,950;

whence it will be eaſy, any part of the circumference being given in degrees and minutes, to aſſign it in parts of the diameter.

Of

---

* The diameter is more nearly to the circumference, as 113 to 355.

Of Surveying and Meaſuring of Land.

HITHERTO we have treated of the meaſuring of angles and ſides, whence it is abundantly eaſy to lay down a field, a plane, or an entire country: For to this nothing is requiſite but the protraction of triangles, and of other plain figures, after having meaſured their ſides and angles. But, as this is eſteemed an important part of Practical Geometry, we ſhall ſubjoin here an account of it, with all poſſible brevity; ſuggeſting withal, that a ſurveyor will improve himſelf more by one day's practice, than by a great deal of reading.

PROP.

PROP. XXIII. PROB.

To Explain what Surveying is, and what Inſtruments Surveyors uſe.

FIRST, it is neceſſary that the ſurveyor view the field that is to be meaſured, and inveſtigate its ſides and angles, by means of an iron chain, (having a particular mark at each foot of length, or at any number of feet, as may be moſt convenient for reducing lines or ſurfaces to the received meaſures*), and the Graphometer deſcribed above. *Secondly*, It is neceſſary to delineate the field *in plano*, or to form a map of it; that is, to lay down, on paper, a figure ſimilar to the field; which is done by the Protractor, (or line of chords), and of the line of equal parts. *Thirdly*, It is neceſſary to find out the area of the field ſo ſurveyed and repreſented by a map. Of this laſt we are to treat below in the ſecond part.

---

* See above, pag. 8. the account of Gunter's chain, and of the chain that is moſt convenient for meaſuring land in Scotland.

The ſides and angles of ſmall fields are ſurveyed by the help of a plain table; which is generally of an oblong rectangular figure, and ſupported by a *fulcrum*, ſo as to turn every way, by means of a ball and ſocket. It is a moveable frame, which ſurrounds the board, and ſerves to keep a clean paper put on the board cloſe and tight to it. The ſides of the frame facing the paper are divided into equal parts every way. The board hath beſides a box with a magnetic needle, and moreover a large index with two ſights. On the edge of the frame of the board are marked degrees and minutes, ſo as to ſupply the room of a Graphometer.

PROP. XXIV.

PROB. FIG. 29.

To delineate a field by the help of a plain table, from one ſtation whence all its angles may be ſeen, and their diſtances meaſured by a chain.

LET the field that is to be laid down be ABCDE. At any convenient place F, let the plain table be erected; cover it with clean paper, in which let ſome point near the middle repreſent the ſtation. Then, applying at this place the index with the ſights, direct it ſo as that through the ſights ſome mark may be ſeen at one of the angles, ſuppoſe A; and from the point F, repreſenting the ſtation, draw a faint right line along the ſide of the index: Then, by the help of the chain, let FA, the diſtance of the ſtation from the foreſaid angle, be meaſured. Then, taking what part you think convenient for a foot or pace from the line of equal parts, ſet off on the faint line the parts correſponding to the line FA that was meaſured; and let there be a mark made repreſenting the angle of the field A. Keeping the table immoveable, the ſame is to be done with the reſt of the angles; then right lines joining thoſe marks ſhall include a figure like to the field, as is evident from 5th, 6. Eucl.

COROLLARY.

The ſame thing is done in like manner by the Graphometer; for having obſerved in each of the triangles, AFB, BFC, CFD, &c. the angle at the ſtation F, and having meaſured the lines from the ſtation to the angles of the field, let ſimilar triangles be protracted on paper, (by the 21. of this), having their common vertex in the point of ſtation. All the lines, excepting thoſe which repreſent the ſides of the field, are to be drawn faint or obſcure.

Note 1. When a ſurveyor wants to lay down a field, let him place diſtinctly in a regiſter all the obſervations of the angles, and the meaſures of the ſides, until, at time and place convenient, he draw out the figure on paper.

Note 2. The obſervations made by the help of the Graphometer are to be examined; for all the angles about the point F ought to be equal to four right ones, by 13th, 1. Eucl.

PROP. XXV.

PROB. FIG. 30.

To lay down a field by means of two ſtations, from each of which all the angles can be ſeen, by meaſuring only the diſtance of the ſtations.

LET the inſtrument be placed at the ſtation F; and having choſen a point repreſenting it upon the paper which is laid upon the plain table, let the index be applied at this point, ſo as to be moveable about it. Then let it be directed ſucceſſively to the ſeveral angles of the field; and, when any angle is ſeen through the ſights, draw an obſcure line along the ſide of the index. Let the index, with the ſights, be directed after the ſame manner to the ſtation G; on the obſcure line, drawn along its ſide, pointing to A, ſet off, from the ſcale of equal parts, a line correſponding to the meaſured diſtance of the ſtations; and this will determine the point G. Then remove the inſtrument to the ſtation G; and applying the index to the line repreſenting the diſtance of the ſtations, place the inſtrument ſo that the firſt ſtation may be ſeen through the ſights. Then the inſtrument remaining immoveable, let the index be applied at the point repreſenting the ſecond ſtation G; and be ſucceſſively directed, by means of its ſights, to all the angles of the field, drawing (as before) obſcure lines; and the interſection of the two obſcure lines that were drawn to the ſame angle from the two ſtations will always repreſent that angle on the plan. Care muſt be taken that thoſe lines be not miſtaken for one another. Lines joining thoſe interſections will form a figure on the paper like to the field.

SCHOLIUM.

It will not be difficult to do the ſame by the graphometer, if you keep a diſtinct account of your obſervations of the angles made by the line joining the ſtations, and the lines drawn from the ſtations to the reſpective angles of the field. And this is the moſt common manner of laying down whole countries. The tops of two mountains are taken for two ſtations, and their diſtance is either meaſured by ſome of the methods mentioned above, or is taken according to common repute. The ſights are ſucceſſively directed towards cities, churches, villages, forts, lakes, turnings of rivers, woods, &c.

Note. The diſtance of the ſtations ought to be great enough, with reſpect to the field that is to be meaſured; ſuch ought to be choſen as are not in a line with any angle of the field. And care ought to be taken likewiſe, that the angles, for example, FAG, FDG, &c. be neither very acute, nor very obtuſe. Such angles are to be avoided as much as poſſible; and this admonition is found very uſeful in practice.

PROP. XXVI.

PROB. FIG. 31.

To lay down any field, however irregular its figure may be, by the help of the Graphometer.

LET ABCEDHG be ſuch a field. Let its angles (in going round it) be obſerved with a graphometer, (by the 12th of this), and noted down; let its ſides be meaſured with a chain; and (by what was ſaid on the 21ſt of this) let a figure, like to the given field, be protracted on paper. If any mountain is in the circumference, the horizontal line hid under it is to be taken for a ſide, which may be found by two or three obſervations, according to ſome of the methods deſcribed above; and its place on the map is to be diſtinguiſhed by a ſhade, that it may be known a mountain is there.

If not only the circumference of the field is to be laid down in the plan, but alſo its contents, as villages, gardens, churches, public roads, we muſt proceed in this manner.

Let there be (for example) a church F, to be laid down in the plan. Let the angles ABF, BAF be obſerved, and protracted on paper in their proper places, the interſection of the two ſides BF and AF will give the place of the church on the paper: Or, more exactly, the lines BF, AF being meaſured, let circles be deſcribed from the centres B and A, with parts from the ſcale correſponding to the diſtances BF and AF, and the place of the church will be at their interſection.

Note 1. While the angles obſerved by the graphometer are taken down, you muſt be careful to diſtinguiſh the external angles, as E and G, that they may be rightly protracted afterwards on paper.

Note 2. Our obſervations of the angles may be examined, by computing, if all the internal angles make twice as many right angles, four excepted, as there are ſides of the figure: For this is demonſtrated by 32d 1. Eucl. But, in place of any external angle DEC, its complement to a circle is to be taken.

PROP. XXVII.

PROB. FIG. 32.

To lay down a plain field without inſtruments.

IF a ſmall field is to be meaſured, and a map of it to be made, and you are not provided with inſtruments; let it be ſuppoſed to be divided into triangles, by right lines, as in the figure; and after meaſuring the three ſides of any of the triangles, for example, of ABC, let its ſides be laid down, from a convenient ſcale, on paper, by the

22d of this. Again, let the other two ſides BD, CD of the triangle CBD be meaſured, and protracted on the paper, by the ſame ſcale as before. In the ſame manner proceed with the reſt of the triangles of which the field is compoſed; and the map of the field will be perfected! For the three ſides of a triangle determine the triangle; whence each triangle on the paper is ſimilar to its correſpondent triangle in the field, and is ſimilarly ſituated: Conſequently, the whole figure is like to the whole field.

SCHOLIUM.

If the field be ſmall, and all its angles may be ſeen from one ſtation, it may be very well laid down by the plain table, by the 24th of this. If the field be larger, and have the requiſite conditions, and great exactneſs is not expected, it likewiſe may be plotted by means of the plain table, or by the Graphometer, according to the 25th of this; but, in fields that are irregular and mountainous, when an exact map is required, we are to make uſe of the Graphometer, as in the 26th of this, but rarely of the plain table.

Having protracted the bounding lines, the particular parts contained within them may be laid down, by the proper operations for this purpoſe, delivered in the 26th propoſition; and the method deſcribed in the 27th propoſition may be ſometimes of ſervice; for we may truſt more to the meaſuring of ſides, than to the obſerving of angles. We are not to compute four-ſided and many-ſided figures, till they are reſolved into triangles: For the ſides do not determine thoſe figures.

In the laying down of cities, or the like, we may make uſe of any of the methods deſcribed above that may be moſt convenient.

The map being finiſhed, it is transferred on clean paper, by putting the firſt ſketch above it, and marking the angles by the point of a ſmall needle. Theſe points being joined by right lines, and the whole illuminated by colours proper to each part, and the figure of the mariner's compaſs being added to diſtinguiſh the north and ſouth, with a ſcale on the margin, the map or plan will be finiſhed and neat.

We have thus briefly and plainly treated of ſurveying, and ſhown by what inſtruments it is performed; having avoided thoſe methods which depend on the magnetic needle, not only becauſe its direction may vary in different places of a field, (the contrary of this at leaſt doth not appear), but becauſe the quantity of an angle obſerved by it cannot be exactly known; for an error of two or three degrees can ſcarcely be avoided in taking angles by it. As for the remaining part of ſurveying, whereby the area of a field already laid down on paper, is found in acres, roods, or any other ſuperficial meaſures; this we leave to the following part, which treats of the menſuration of ſurfaces.

‘ Beſides the inſtruments deſcribed above, ‘ a ſurveyor ought to be provided with an

‘ off-ſet ſtaff, equal in length to ten links of ‘ the chain, and divided into ten equal parts. ‘ He ought likewiſe to have ten arrows or ‘ ſmall ſtraight ſticks, near two feet long, ‘ ſhod with iron ferrils. When the chain is ‘ firſt opened, it ought to be examined by ‘ the off-ſet ſtaff. In meaſuring any line, ‘ the leader of the chain is to have the ten ‘ arrows at firſt ſetting out. When the ‘ chain is ſtretched in the line, and the near ‘ end touches the place from which you ‘ meaſure, the leader ſticks one of the ten ‘ arrows in the ground, at the far end of ‘ the chain. Then the leader leaving the ‘ arrow, proceeds with the chain another ‘ length; and the chain being ſtretched in ‘ the line, ſo that the near end touches the ‘ firſt arrow, the leader ſticks down another ‘ arrow at his end of the chain. The line ‘ is preſerved ſtraight, if the arrows be always ſet ſo as to be in a right line with ‘ the place you meaſure from, and that to ‘ which you are going. In this manner ‘ they proceed till the leader have no more 'arrows. At the eleventh chain, the arrows are to be carried to him again, and 'he is to ſtick one of them in the ground, 'at the end of the chain. And the ſame is 'to be done at the 21 31. 41. &c. chains, 'if there are ſo many in the right line to be 'meaſured. In this manner you can hardly 'commit an error in numbering the chains, 'unleſs of ten chains at once.

'The off-ſet ſtaff ſerves for meaſuring 'readily the diſtances of any things proper 'to be repreſented in your plan, from the 'ſtation-line while you go along. Theſe 'diſtances ought to be entered into your 'field-book, with the correſponding diſtances 'from the laſt ſtation, and proper remarks, 'that you may be enabled to plot them 'juſtly, and be in no danger of miſtaking 'one for another, when you extend your 'plan. The field-book may be conveni'ently divided into five columns. In the 'middle column the angles at the ſeveral 'ſtations taken by the Theodolite are to be 'entered, with the diſtances from the ſta-

'tions. The diſtances taken by the off-ſet 'ſtaff, on either ſide of the ſtation-line, are 'to be entered into columns, on either ſide 'of the middle column, according to their 'poſition with reſpect to that line. The 'names or characters of the objects, with 'proper remarks, may be entered in co'lumns on either ſide of theſe laſt.

'Becauſe, in the place of the Grapho'meter deſcribed by our author, Surveyors 'now make uſe of the Theodolite, we ſhall 'ſubjoin a deſcription of Mr Siſſon's lateſt 'improved Theodolite from Mr Gardiner's 'Practical Surveying improved. See a fi'gure of it in plate 4.

'In this inſtrument, the three ſtaffs, by 'braſs ferrils at top, ſcrew into bell-metal 'joints, that are moveable between braſs 'pillars, fixed on a ſtrong braſs plate; in 'which, round the centre, is fixed a ſoc'ket with a ball moveable in it, and upon 'which the four ſcrews preſs, that ſet the 'limb horizontal: Next above is another 'ſuch plate, through which the ſaid ſcrews 'paſs, and on which, round the centre, is 'fixed a fruſtum of a cone of bell-metal, 'whoſe axis (being connected with the 'centre of the ball) is always perpendicular 'to the limb, by means of a conical braſs 'ferril fitted to it, whereon is fixed the 'compaſs-box; and on it the limb, which 'is a ſtrong bell-metal ring, whereon are 'moveable three braſs indexes; in whoſe 'plate are fixed four braſs pillars, that, 'joining at top, hold the centre-pin of the 'bell-metal double ſextant, whoſe double 'index is fixed on the centre of the ſame 'plate: Within the double ſextant is fixed 'the ſpirit-level, and over it the teleſcope.

'The compaſs-box is graved with two 'diamonds for north and ſouth, and with '20 degrees on both ſides of each, that the 'needle may be ſet to the variation, and its 'error alſo known.

'The limb is two *Fleurs de luce* againſt 'the diamonds in the box, inſtead of 180 'each; and is curiouſly divided into whole 'degrees, and numbered to the left hand 'at every ten to twice 180, having three

‘ indexes diſtant 120, (with Nonius's diviſions on each for the decimals of a degree), that are moved by a pinion fixed below one of them, without moving the limb; and in another is a ſcrew and ſpring under, to fix it to any part of the limb. It has alſo diviſions numbered, for taking the quarter girt in inches of round timber at the middle height, when ſtanding ten feet horizontally diſtant from its centre; which at 20 muſt be doubled, and at 30 tripled; to which a ſhorter index is uſed, having Nonius's diviſions for the decimals of an inch; but an abatement muſt be made for the bark, if not taken off.

‘ The double ſextant is divided on one ſide from under its centre (when the ſpirit-tube and teleſcope are level) to above 60 degrees each way, and numbered at 10. 20. &c. and the double index (through which it is moveable) ſhews on the ſame ſide the degree and decimal of any altitude or depreſſion to that extent by Nonius's diviſions: On the other ſide are di-

'viſions numbered for taking the upright 'height of timber, &c. in feet, when diſtant '10 feet; which at 20 muſt be doubled, 'and at 30 tripled; and alſo the quantities 'for reducing hypothenuſal lines to horizontal. It is moveable by a pinion fixed 'in the double index.

'The teleſcope is a little ſhorter than 'the diameter of the limb, that a fall may 'not hurt it; yet it will magnify as much, 'and ſhew a diſtant object as perfect, as 'moſt of triple its length. In its focus are 'very fine croſs wires, whoſe interſection 'is in the plane of the double ſextant; and 'this was a whole circle, and turned in a 'lathe to a true plane, and is fixed at right 'angles to the limb; ſo that, whenever the 'limb is ſet horizontal, (which is readily 'done by making the ſpirit-tube level over 'two ſcrews, and the like over the other 'two), the double ſextant and teleſcope are 'moveable in a vertical plane; and then 'every angle taken on the limb (though 'the teleſcope be never ſo much elevated

‘ or depreſſed) will be an angle in the plane ‘ of the horizon. And this is abſolutely ‘ neceſſary in plotting a horizontal plane.

‘ If the lands to be plotted are hilly, and ‘ not in any one plane, the lines meaſured ‘ cannot be truly laid down on paper, without being reduced to one plane, which ‘ muſt be the horizontal, becauſe angles are ‘ taken in that plane.—

‘ In viewing my objects, if they have ‘ much altitude or depreſſion, I either write ‘ down the degree and decimal ſhewn on ‘ the double ſextant, or the links ſhewn on ‘ the back-ſide; which laſt ſubtracted from ‘ every chain in the ſtation-line, leaves the ‘ length in the horizontal plane. But, if ‘ the degree is taken, the following table ‘ will ſhew the quantity.

A TABLE of the links to be ſubtracted out of every chain in hypothenuſal lines *of ſeveral degrees altitude, or depreſſion, for reducing them to* horizontal.

Degrees.	*Links.*	*Degrees.*	*Links.*	*Degrees.*	*Links.*
4,05	—$\frac{1}{4}$	14,07	—3	23,074	—8
5,73	—$\frac{1}{2}$	16,26	—4	24,495	—9
7,02	—$\frac{3}{4}$	18,195	—5	25,84	—10
8,11	—1	19,95	—6	27,13	—11
11,48	—2	21,565	—7	28,36	—12

‘ Let the firſt ſtation-line really meaſure ‘ 1107 links, and the angle of altitude or ‘ depreſſion be 19°, 95; looking in the table ‘ I find againſt 19°, 95, is ſix links. Now ‘ 6 times 11 is 66; which ſubtracted from ‘ 1107, leaves 1041, the true length to be ‘ laid down in the plan.

‘ It is uſeful in ſurveying, to take the ‘ angles, which the bounding lines form, ‘ with the magnetic needle, in order to check ‘ the angles of the figure, and to plot them ‘ conveniently afterwards.’

PART

PART II.

Of the Surfaces of Bodies.

THE ſmalleſt ſuperficial meaſure with us is a ſquare inch; 144 of which makes a ſquare foot. Wrights make uſe of theſe in the meaſuring of deals and planks; but the ſquare foot which the glaziers uſe in meaſuring of glaſs, conſiſts only of 64 ſquare inches. The other meaſures are, *firſt*, the ell ſquare; 2*dly*, the fall, containing 36 ſquare ells; 3*dly*, the rood, containing 40 falls; 4*thly*, the acre, containing 4 roods. Slaters, maſons, and pavers, uſe the ell ſquare and the fall; ſurveyors of land uſe the ſquare ell, the fall, the rood, and the acre.

The ſuperficial meaſures of the Engliſh, are *firſt*, the ſquare foot; 2*dly*, the ſquare yard, containing 9 ſquare feet; for their yard contains only 3 feet; 3*dly*, the pole containing 30¼ ſquare yards; 4*thly*, the rood, containing 40 poles; *5thly*, the acre, containing 4 roods. And hence it is eaſy to reduce our ſuperficial meaſures to the Engliſh, or theirs to our.

'In order to find the content of a field, 'it is moſt convenient to meaſure the lines 'by the chains deſcribed above, p. 4. that 'of 22 yards for computing the Engliſh 'acres, and that of 24 Scots ells for the 'acres of Scotland. The chain is divided 'into 100 links, and the ſquare of the chain 'is 10,000 ſquare links; ten ſquares of the 'chain, or 100,000 ſquare links give an 'acre. Therefore, if the area be expreſſed 'by ſquare links, divided by 100,000, or 'cut off five decimal places, and the quo'tient ſhall give the area in acres and de'cimals of an acre. Write the entire acres 'apart; but multiply the decimals of an 'acre by 4, and the product ſhall give the 'remainder of the area in roods and deci'mals of a rood. Let the entire roods be 'noted apart after the acres; then multiply 'the decimals of a rood by 40, and the 'product ſhall give the remainder of the 'area in falls or poles. Let the entire falls 'or poles be then writ after the roods, and 'multiply the decimals of a fall by 36, if 'the area is required in the meaſures of 'Scotland; but multiply the decimals of a 'pole by 30¼, if the area is required in the 'meaſures of England, and the product 'ſhall give the remainder of the area in 'ſquare ells in the former caſe, but in ſquare 'yards in the latter. If, in the former caſe, 'you would reduce the decimals of the 'ſquare ell to ſquare feet, multiply them by '9.50694; but, in the latter caſe, the de'cimals of the Engliſh ſquare yard are re'duced to ſquare feet, by multiplying them 'by 9.

'Suppoſe, for example, that the area ap'pears to contain 12.65842 ſquare links of 'the chain of 24 ells; and that this area is 'to be expreſſed in acres, roods, falls, &c. 'of the meaſures of Scotland. Divide the 'ſquare links by 100,000, and the quotient '12.65842 ſhows the area to contain 12

‘ acres $\frac{65842}{100000}$ of an acre. Multiply the decimal part by 4, and the product 2.63368 ‘ gives the remainder in roods and decimals ‘ of a rood. Thoſe decimals of the rood ‘ being multiplied by 40, the product gives ‘ 25.3472 falls. Multiply the decimals of ‘ the fall by 36, and the product gives ‘ 12.4992 ſquare ells. The decimals of ‘ the ſquare ell multiplied by 9.50994 give ‘ 4.7458 ſquare feet. Therefore the area ‘ propoſed amounts to 12 acres, 2 roods, 25 ‘ falls, 12 ſquare ells, and 4 $\frac{7458}{10000}$ ſquare ‘ feet.

‘ But, if the area contains the ſame ‘ number of ſquare links of Gunter’s chain, ‘ and is to be expreſſed by Engliſh meaſures, the acres and roods are computed ‘ in the ſame manner as in the former caſe. ‘ The poles are computed as the falls. But ‘ the decimals of the pole, viz. $\frac{5472}{10000}$, are ‘ to be multiplied by $30\frac{1}{4}$ (or 30.25), and ‘ the product gives 10.5028 ſquare yards. ‘ The decimals of the ſquare yard, multiplied by 9, give 4.5252 ſquare feet;

‘ therefore, in this caſe, the area is in English measure 12 acres, 2 roods, 25 poles, ‘ 10 ſquare yards, and 4 $\frac{5252}{10000}$ ſquare feet.

‘ The Scots acre is to the Engliſh acre, ‘ by ſtatute, as 100,000 to 78,694, if we ‘ have regard to the difference betwixt the ‘ Scots and Engliſh foot above mentioned. ‘ But it is cuſtomary in ſome parts of England to have 18,21, &c. feet to a pole, ‘ and 160 ſuch poles to an acre; whereas, ‘ by the ſtatute, 16$\frac{1}{2}$ feet make a pole. In ‘ ſuch caſes the acre is greater in the duplicate ratio of the number of feet to a pole.

‘ They who meaſure land in Scotland by ‘ an ell of 37 Engliſh inches, make the acre ‘ leſs than the true Scots acre by 593$\frac{6}{10}$ ‘ ſquare Engliſh feet, or by about $\frac{1}{93}$ of the ‘ acre.

‘ An huſband-land contains 6 acres of ‘ ſock and ſcythe-land, that is, of land that ‘ may be tilled with a plough, and mown ‘ with a ſcythe; 13 acres of arable land ‘ make an oxgang or oxengate; four oxengate make a pound-land of old extent (by

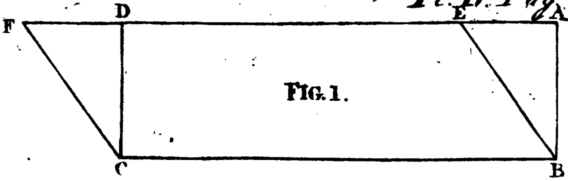
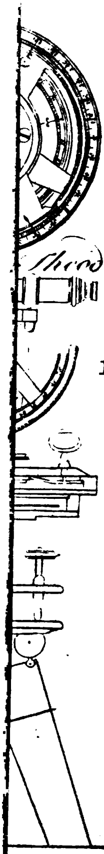


FIG. 1.



Theodolite.

FIG. 2.

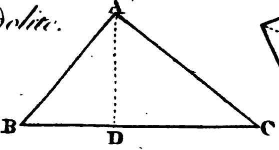


FIG. 3.

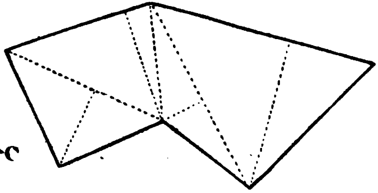


FIG. 4.

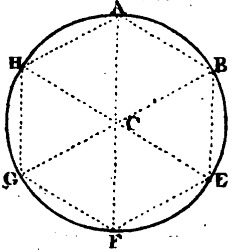


FIG. 5.

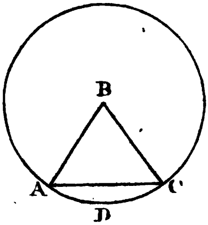


FIG. 6.

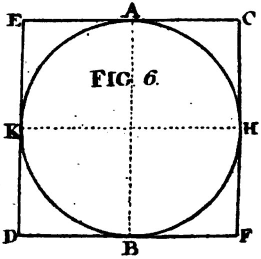


FIG. 7.

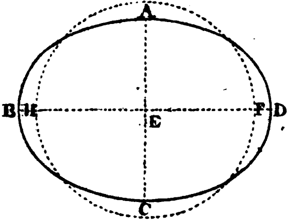


FIG. 10.

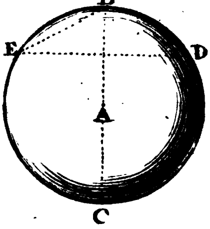


FIG. 9.

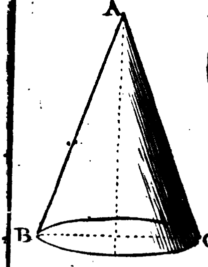
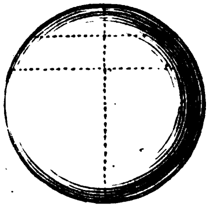


FIG. 11.



UNIV. OF MICH.

‘a decree of the exchequer, March 11. 1585), and is called *librato terrae*. A forty ſhilling land of old extent contains eight oxgang, or 104 acres.

‘The Arpent about Paris contains 32400 ſquare Paris feet, and is equal to $2\frac{2}{3}$ Scots roods, or $3\frac{37}{100}$ Engliſh roods.

‘The *Actus quadratus*, according to Varro, Collumella, &c. was a ſquare of 120 Roman feet. The jugerum was the double of this. ’Tis to the Scots acre as 10,000 to 20,456, and to the Engliſh acre as 10,000 to 16,097. It was divided (like the As) into 12 unciae, and the uncia into 24 ſcrupula.’ This, with the three preceding paragraphs, are taken from an ingenious manuſcript written by Sir Robert Stewart profeſſor of natural philoſophy. The greateſt part of the table in p. 6. was taken from it likewiſe.

PROP.

PROP. I.

PROB. FIG. 1.

To find out the area of a rectangular parallelogram ABCD.

LET the ſide AB, for example, be 5 feet long, and BC (which conſtitutes with AB a right angle at B) be 17 feet. Let 17 be multiplied by 5, and the product 85 will be the number of ſquare feet in the area of the figure ABCD. But, if the parallelogram propoſed is not rectangular, as BEFC, its baſe BC multiplied into its perpendicular height AB (not into its ſide BE) will give its area. This is evident from 35th 1. Eucl.

PROP.

PROP. II.

PROB. FIG. 2.

To find the Area of a given Triangle.

LET the triangle BAC be given, whoſe baſe BC is ſuppoſed 9 feet long: Let the perpendicular AD be drawn from the angle A oppoſite to the baſe, and let us ſuppoſe AD to be four feet. Let the half of the perpendicular be multiplied into the baſe, or the half of the baſe into the perpendicular, or take the half of the product of the whole baſe into the perpendicular, the product gives 18 ſquare feet for the area of the given triangle.

But, if only the ſides are given, the perpendicular is found either by protracting the triangle, or by 12th and 13th 2. Eucl. or by trigonometry. But how the area of a triangle may be found from the given ſides only, ſhall be ſhewn in the 4th prop. of this part,

PROP. III.

PROB. FIG. 3.

To find the Area of any rectilineal Figure.

IF the figure be irregular, let it be reſolved into triangles ; and drawing perpendiculars to the baſes in each of them, let the area of each triangle be found by the preceding Prop. and the ſum of theſe areas will give the area of the figure.

SCHOLIUM 1.

In meaſuring boards, planks, and glaſs, their ſides are to be meaſured by a foot-rule divided into 100 equal parts ; and after multiplying the ſides, the decimal fractions are eaſily reduced to leſſer denominations. The menſuration of theſe is eaſy, when they are rectangular parallelograms.

SCHOLIUM 2.

If a field is to be meaſured, let it firſt be plotted on paper, by ſome of the methods deſcribed in the preceding part, and let the figure ſo laid down be divided into triangles, as was ſhown in the preceding propoſition.

The baſe of any triangle, or the perpendicular upon the baſe, or the diſtance of any two points of the field, is meaſured by applying it to the ſcale according to which the map is drawn.

SCHOLIUM 3.

But, if the field given be not in a horizontal plane, but uneven and montainous, the ſcale gives the horizontal line between any two points, but not their diſtance meaſured on the uneven ſurface of the field. And indeed it would appear that the horizontal plane is to be accounted the area of an uneven and hilly country. For, if ſuch ground is laid out for building on, or for planting with trees, or bearing corn, ſince theſe ſtand perpendicular to the horizon, it is plain that a mountainous country cannot be conſidered as of greater extent for thoſe uſes than the horizontal plane; nay, perhaps, for nouriſhing of plants, the horizontal plane may be preferable.

If, however, the area of a figure, as it lies irregularly on the ſurface of the earth, is to be meaſured, this may be eaſily done by reſolving it into triangles as it lies. The ſum of their areas will be the area ſought; which exceeds the area of the horizontal figure more or leſs, according as the field is more or leſs uneven.

PROP. IV.

PROB. FIG. 2.

The ſides of a triangle being given, to find the area, without finding the perpendicular.

LET all the ſides of the triangle be collected into one ſum; from the half of which let the ſides be ſeparately ſubtracted, that three differences may be found betwixt the foreſaid half ſum and each ſide; then let theſe three differences and the half ſum be multiplied into one another, and the ſquare root of the product will give the area of the triangle. For example, let the ſides be 10, 17, 21; the half of their ſum is 24; the three differences betwixt this half ſum and the three ſides, are 14, 7, and 3. The firſt being multiplied by the ſecond, and their product by the third, we have 294 for the product of the differences; which multiplied by the foreſaid half ſum 24, gives 7056; the ſquare root of which 84 is the area of the triangle. The demonſtration of this, for the ſake of brevity, we omit. It is to be found in ſeveral treatiſes, particularly in Clavius's Practical Geometry.

PROP. V.

THEOR. FIG. 4.

The area of the ordinate figure ABEFGH *is equal to the product of the half circumference of the polygon, multiplied into the perpendicular drawn from the centre of the circumſcribed circle to the ſide of the polygon.*

FOR the ordinate figure can be reſolved into as many equal triangles, as there are ſides of the figure; and, ſince each triangle is equal to the product of half the baſe into the perpendicular, it is evident that the ſum of all the triangles together, that is, the polygon, is equal to the product of half the ſum of the baſes (that is, the half of the circumference of the polygon) into the common perpendicular height of the triangles drawn from the centre C to one of the ſides; for example, to AB.

PROP. VI.

PROB. FIG. 5.

The area of a circle is found by multiplying the half of the periphery into the radius, or the half of the radius into the periphery.

FOR a circle is not different from an ordinate or regular polygon of an infinite number of ſides, and the common height of the triangles into which the polygon or circle may be ſuppoſed to be divided is the radius of the circle.

Were it worth while, it were eaſy to demonſtrate accurately this propoſition, by means of the inſcribed and circumſcribed figures, as is done in the 5th Prop. of the treatiſe of Archimedes concerning the dimenſions of the circle.

COROLLARY.

Hence alſo it appears that the area of the ſector ABCD is produced, by multiplying the half of the arch into the radius; and likewiſe that the area of the ſegment of the circle ADC is found, by ſubtracting from the area of the ſector the area of the triangle ABC.

PROP. VII.

THEOR. FIG. 6.

The circle is to the ſquare of the diameter, as 11 *to* 14 *nearly.*

FOR, if the diameter AB be ſuppoſed to be 7, the circumference AHBK will be almoſt 22 (by the 22d Prop. of the firſt part of this); and the area of the ſquare DC will be 49; and, by the preceding Prop. of this, the area of the circle will be $38\frac{1}{2}$: Therefore the ſquare DC will be to the inſcribed circle as 49 to $38\frac{1}{2}$, or as 98 to 77, that is, as 14 to 11. *Q. E. D.*

If greater exactneſs is required, you may proceed to any degree of accuracy: For the ſquare DC is to the inſcribed circle, as 1 to $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$, &c. *in infinitum*,

'This ſeries will be of no ſervice for 'computing the area of the circle accurately, 'without ſome further artifice, becauſe it 'converges at too ſlow a rate. The area 'of the circle will be found exactly enough 'for moſt purpoſes, by multiplying the 'ſquare of the diameter by 7854, and divi'ding by 10,000, or cutting off four deci'mal places from the product; for the area 'of the circle is to the circumſcribed ſquare 'nearly as 7854 to 10,000.'

PROP.

PROP. VIII.

PROB. FIG. 7.

To find the Area of a given Ellipſe.

LET ABCD be an ellipſe, whoſe greater diameter is BD, and leſſer AC, biſecting the greater perpendicularly in E. Let a mean proportional HF be found (by 13th 6. Eucl.) between AC and BD, and (by the 6th of this) find the area of the circle deſcribed on the diameter HF. I ſay, that this area is equal to the area of the ellipſe ABCD. For becauſe, as BD to AC, ſo the ſquare of BD to the ſquare of HF, (by 2. Cor. 20th 6. Eucl.): But, (by the 2d 12. Eucl.) as the ſquare of BD to the ſquare of HF, ſo is the circle of the diameter BD to the circle of the diameter HF: Therefore, as BD to AC, ſo is the circle of the diameter BD to the circle of the diameter HF. And (by the 5th Prop. of Archimedes of ſpheroids) as the greater diameter BD to the leſſer AC, ſo is the circle of the diameter BD to the ellipſe ABCD. Conſequently (by the 11th 5. Eucl.) the circle of the diameter BD will have the ſame proportion to the circle of the diameter HF, and to the ellipſe ABCD. Therefore, by 9th 5. Eucl. the area of the circle of the diameter HF will be equal to the area of the ellipſe ABCD. *Q. E. D.*

SCHOLIUM.

From this and the two preceding propoſitions, a method is derived of finding the area of an ellipſe. There are two ways: 1ſt, Say, as one is to the leſſer diameter, ſo is the greater diameter to a fourth number, (which is found by the rule of three.) Then again ſay, as 14 to 11, ſo is the 4th number found to the area ſought. But the ſecond way is ſhorter. Multiply the leſſer diameter into the greater, and the product by 11; then divide the whole product by 14, and the quotient will be the area ſought of the ellipſe. For example, Let the greater diameter be 10, and the leſſer 7, by multiplying 10 by 7, the product is 70; and multiplying that by 11, it is 770; and dividing 770 by 14, the quotient will be 55, which is the area of the ellipſe ſought.

'The area of the ellipſe will be found 'more accurately, by multiplying the pro'duct of the two diameters by 7854.'

We ſhall add no more about other plain ſurfaces, whether rectilinear or curvilinear, which ſeldom occur in practice; but ſhall ſubjoin ſome propoſitions about meaſuring the ſurfaces of ſolids.

PROP. IX. PROB.

To meaſure the ſurface of any Priſm.

BY the 14th definition of the 11th Eucl. a priſm is contained by planes, of which two oppoſite ſides (commonly called the baſes) are plain rectilineal figures; which are either regular and ordinate, and meaſured by Prop. 5. of this part; or however irregular, and then they are meaſured by the 3d Prop. of this book. The other ſides are parallelograms, which are meaſured by the 1ſt Prop. of the ſecond part; and the whole ſuperficies of the priſm conſiſts of the ſum of thoſe taken altogether.

PROP. X. PROB.

To meaſure the ſuperficies of any Pyramid.

SINCE its baſis is a rectilineal figure, and the reſt of the planes terminating in the top of the pyramid are triangles;

theſe meaſured ſeparately, and added together, give the ſurface of the pyramid required.

PROP. XI. PROB.

To meaſure the ſuperficies of any Regular Body.

THOSE bodies are called regular, which are bounded by equilateral and equiangular figures. The ſuperficies of the tetraedron conſiſts of four equal and equiangular triangles; the ſuperficies of the hexaedron, or cube, of ſix equal ſquares; an octaedron, of eight equal equilateral triangles; a dodecaedron, of twelve equal and ordinate pentagons; and the ſuperficies of an icoſiaedron, of twenty equal and equilateral triangles. Therefore it will be eaſy to meaſure theſe ſurfaces from what has been already ſhown.

In the ſame manner we may meaſure the ſuperficies of a ſolid contained by any planes.

PROP. XII.

PROB. FIG. 8.

To meaſure the Superficies of a Cylinder.

BECAUSE a cylinder differs very little from a priſm, whoſe oppoſite planes (or baſes) are ordinate figures of an infinite number of ſides, it appears that the ſuperficies of a cylinder, without the baſes, is equal to an infinite number of parallelograms; the common altitude of all which is the height of the cylinder, and the baſis of them all differ very little from the periphery of the circle, which is the baſe of the cylinder. Therefore this periphery multiplied into the common height, gives the ſuperficies of the cylinder, excluding the baſes; which are to be meaſured ſeparately by the help of the 6th Prop. of this part.

This propoſition concerning the meaſure of the ſurface of the cylinder (excluding its baſis) is evident from this, That, when it is conceived to be ſpread out, it becomes a parallelogram, whoſe baſe is the periphery of the circle of the baſe of the cylinder ſtretched into a right line, and whoſe height is the ſame with the height of the cylinder.

PROP. XIII.

PROB. FIG. 9.

To meaſure the Surface of a Right Cone.

THE ſurface of a right cone is very little different from the ſurface of a right pyramid, having an ordinate polygon for its baſe, of an infinite number of ſides; the ſurface of which (excluding the baſe) is equal to the ſum of the triangles. The ſum of the baſes of theſe triangles is equal to the periphery of the circle of the baſe, and the common height of the triangles is the ſide of the cone AB: Wherefore the ſum of theſe triangles is equal to the product of the ſum of the baſes (*i. e.* the periphery of the baſe of the cone) multiplied into the half of the common height, or it is equal to the product of the periphery of the baſe.

If the area of the baſes is likewiſe wanted, it is to be found ſeparately by the 6th Prop. of this part. If the ſurface of a cone is ſuppoſed to be ſpread out on a plane, it will become a ſector of a circle, whoſe radius is the ſide of the cone; and the arch terminating the ſector is made from the periphery of the baſe. Whence, by Corol. 6. Prop. of this, its dimenſion may be found.

COROLLARY.

Hence it will be eaſy to meaſure the ſurface of a *fruſtum* of a cone cut by a plane parallel to the baſe. As to what relates to the meaſuring of the ſurface of the ſcalenous cone, becauſe it is not very uſeful in practice, we ſhall not deſcribe the method;

which would carry us beyond the limits of this treatiſe.

PROP. XIV.

PROB. FIG. 10.

To meaſure the Surface of a given Sphere.

LET there be a ſphere, whoſe centre is A, and let the area of its convex ſurface be required. Archimedes demonſtrates (37. Prop. 1. book of the ſphere and cylinder) that its ſurface is equal to the area of four great circles of the ſphere; that is, let the area of the great circle be multiplied by 4, and the product will give the area of the ſphere; or, by the 20th 6. and 2d 12. of Eucl. the area of the ſphere given is equal to the area of a circle whoſe radius is the right line BC, the diameter of the ſphere. Therefore, having meaſured (by 6th Prop. of this part) the circle deſcribed with the radius BC, this will give the ſurface of the ſphere.

PROP.

PROP. XV.

PROB. FIG. 10.

To meaſure the ſurface of a ſegment of a Sphere.

LET there be a ſegment cut off by the plane ED. Archimedes demonſtrates (49. and 50. 1. *de ſphaera*) that the ſurface of this ſegment, excluding the circular baſe, is equal to the area of a circle whoſe radius is the right line BE drawn from the vertex B of the ſegment to the periphery of the circle DE. Therefore, by the 6th Prop. of this part, it is eaſily meaſured.

COROLLARY 1.

Hence that part of the ſurface of a ſphere that lieth between two parallel planes is eaſily meaſured, by ſubtracting the ſurface of the leſſer ſegment from the ſurface of the greater ſegment.

COROL-

COROLLARY 2.

Hence likewiſe it follows, that the ſurface of a cylinder, deſcribed about a ſphere (excluding the baſis) is equal to the ſurface of the ſphere, and the parts of the one to the parts of the other, intercepted between planes parallel to the baſis of the cylinder.

PART III.

Of Solid Figures, and their Menſuration.

AS in the preceding parts we took an inch for the ſmalleſt meaſure in length, and an inch ſquare for the ſmalleſt ſuperficial meaſure; ſo now, in treating of the menſuration of ſolids, we take a cubical inch for the ſmalleſt ſolid meaſure. Of theſe 109 makes a Scots pint; other liquid meaſure depends on this, as is generally known.

In dry meaſures, the firlot, by ſtatute, contains 19$\frac{1}{2}$ pints; and on this depend the other

FIG. 2.

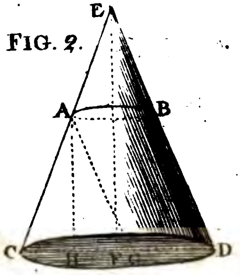


FIG. 3.

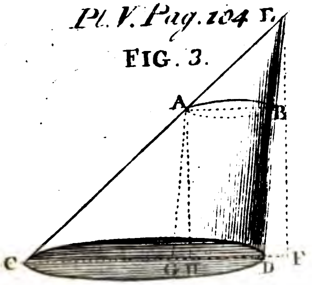


FIG. 4.

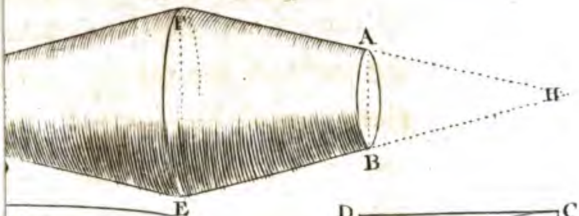
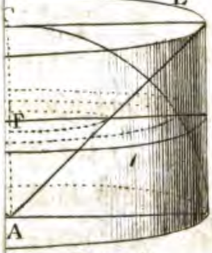


FIG. 5.



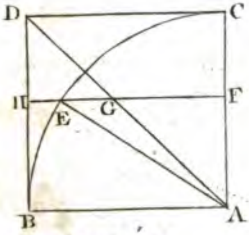


FIG. 7.

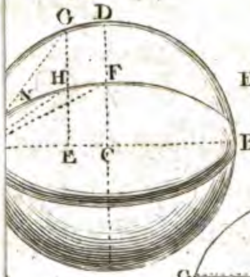
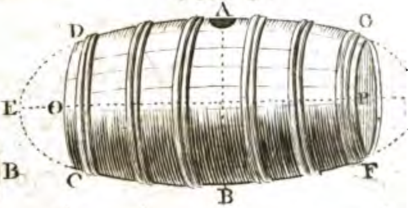
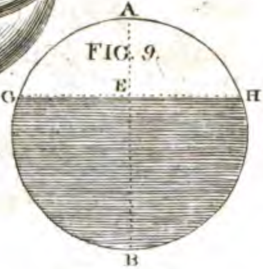
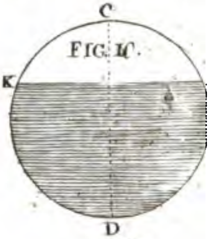


FIG. 8.







UNIV. OF MICH.

other dry meaſures: Therefore, if the content of any ſolid be given in cubical inches, it will be eaſy to reduce the ſame to the common liquid or dry meaſures, and, converſely, to reduce theſe to ſolid inches. The liquid and dry meaſures in uſe among other nations, are known from their writers.

‘ As to the Engliſh liquid meaſures, by
‘ act of parliament 1706, any round veſſel,
‘ commonly called a cylinder, having an
‘ even bottom, being ſeven inches in diameter
‘ throughout, and ſix inches deep from
‘ the top of the inſide to the bottom (which
‘ veſſel will be found, by computation, to
‘ contain $230\frac{907}{1000}$ cubical inches); or any
‘ veſſel containing 231 cubical inches, and
‘ no more, is deemed to be a lawful wine-
‘ gallon. An Engliſh pint therefore con-
‘ tains $28\frac{7}{8}$ cubical inches; two pints makes
‘ a quart; four quarts a gallon; 18 gallons
‘ a roundlet; three roundlets and an half,
‘ or 63 gallons, make a hogſhead; the half
‘ of a hogſhead is a barrel; one hogſhead

‘ and a third, or 84 gallons, make a puncheon; one puncheon and a half, or two ‘ hogſheads, or 126 gallons, make a pipe ‘ or butt; the third part of a pipe, or 42 ‘ gallons, make a tierce; two pipes, or three ‘ puncheons, or four hogſheads, make a ton ‘ of wine. Though the Engliſh wine gallon is now fixed at 231 cubical inches, ‘ the ſtandard kept at Guildhall being meaſured, before many perſons of diſtinction, ‘ May 25. 1688, it was found to contain ‘ only 224 ſuch inches.

‘ In the Engliſh beer meaſure, a gallon ‘ contains 282 cubical inches; conſequently ‘ $35\frac{1}{4}$ cubical inches make a pint, two pints ‘ make a quart, four quarts make a gallon, ‘ nine gallons a firkin, four firkins a barrel. ‘ In ale, eight gallons make a firkin, and 32 ‘ gallons make a barrel. By an act of the ‘ firſt of William and Mary, 34 gallons is ‘ the barrel, both for beer and ale, in all ‘ places, except within the weekly bills of ‘ mortality.

‘ In Scotland, it is known that four gills ‘ make a mutchkin, two mutchkins make ‘ chopin, a pint is two chopins, a quart is ‘ two pints, and a gallon is four quarts or ‘ eight pints. The accounts of the cubical ‘ inches contained in the Scots pint vary ‘ conſiderably from each other. According ‘ to our author, it contains 109 cubical ‘ inches. But the ſtandard-jugs kept by ‘ the Dean-of-guild of Edinburgh (one of ‘ which has the year 1555, with the arms ‘ of Scotland, and the town of Edinburgh, ‘ marked upon it) having been carefully ‘ meaſured ſeveral times, and by different ‘ perſons, the Scots pint, according to thoſe ‘ ſtandards, was found to contain about ‘ $103\frac{4}{10}$ cubic inches. The pewterers jugs ‘ (by which the veſſels in common uſe are ‘ made) are ſaid to contain ſometimes be‘twixt 105 and 106 cubic inches. A caſk ‘ that was meaſured by the brewers of E‘dinburgh, before the Commiſſioners of ‘ Exciſe in 1707, was found to contain $46\frac{7}{8}$ ‘ Scots pints; the ſame veſſel contained $18\frac{1}{2}$

‘ Engliſh ale-gallons. Suppoſing this menſurating to be juſt, the Scots pint will be ‘ to the Engliſh ale-gallon as 289 to 750; ‘ and, if the Engliſh ale-gallon be ſuppoſed ‘ to contain 282 cubical inches, the Scots ‘ pint will contain 108.664 cubical inches. ‘ But it is ſuſpected, on ſeveral grounds, ‘ that this experiment was not made with ‘ ſufficient care and exactneſs.

‘ The Commiſſioners appointed by authority of Parliament to ſettle the meaſures ‘ and weights, in their act of February 19. ‘ 1618, relate, That, having cauſed fill the ‘ Linlithgow firlot with water, they found ‘ that it contained 21¼ pints of the juſt ‘ Stirling jug and meaſure. They likewiſe ‘ ordain, that this ſhall be the juſt and only ‘ firlot, and add, *That the wideneſs and* ‘ *breadneſs of the which firlot, under and* ‘ *above, even over within the buirds, ſhall* ‘ *contain nineteen inches and the ſixth part* ‘ *of an inch, and the deepneſs ſeven inches and* ‘ *a third part of an inch.* According to this ‘ act (ſuppoſing their experiment and com-

‘ putation to have been accurate) the pint ‘ contained only 99.56 cubical inches; for ‘ the content of ſuch a veſſel as is deſcribed ‘ in the act, is 2115.85, and this divided by ‘ 21¼, gives 99.56. But, by the weight of ‘ water ſaid to fill this firlot in the ſame act, ‘ the meaſure of the pint agrees nearly with ‘ the Edinburgh ſtandard above mentioned.

‘ As for the Engliſh meaſures of corn, ‘ the Wincheſter gallon contains 272¼ cubical inches, two gallons make a peck, four ‘ pecks, or eight gallons, (that is, 2178 cubical inches) make a buſhel, and a quarter ‘ is eight buſhels.

‘ Our author ſays, that 19½ Scots pints ‘ make a firlot. But this does not appear to ‘ be agreeable to the ſtatute above mentioned, nor to the ſtandard-jugs. It may be ‘ conjectured that the proportion aſſigned ‘ by him has been deduced from ſome experiment of how many pints, according ‘ to common uſe, were contained in the ‘ firlot. For, if we ſuppoſe thoſe pints to ‘ have been each of 108.664 cubical inches,

‘ according to the experiment made in the ‘ 1707 before the Commiſſioners of Exciſe, ‘ deſcribed above; then $19\frac{1}{3}$ ſuch pints will ‘ amount to 2118.94 cubical inches, which ‘ agrees nearly with 2115.85, the meaſure ‘ of the firlot by ſtatute above mentioned. ‘ But it is probable, that in this he follow‘ed the act 1587, where it is ordained, ‘ That the wheat-firlot ſhall contain 19 ‘ pints and two joucattes. A wheat-firlot, ‘ marked with the Linlithgow ſtamps, be‘ing meaſured, was found to contain about ‘ 2211 cubical inches. By the ſtatute of ‘ 1618 the barley-firlot was to contain 31 ‘ pints of the juſt Stirling jug.

‘ A Paris pint is 48 cubical Paris inches, ‘ and is nearly equal to an Engliſh wine ‘ quart. The *Boiſſean* contains 644.68099 ‘ Paris cubical inches, or 780.36 Engliſh ‘ cubical inches.

‘ The Roman *Amphora* was a cubical ‘ Roman foot, the *Congius* was the eighth ‘ part of the *Amphora*, the *Sextarius* was ‘ one ſixth of the *Congius*. They divided

‘ the *Sextarius* like the *As* or *Libra.* Of ‘ dry meaſures the *Medimnus* was equal to ‘ two *Amphoras*, that is, about $1\frac{1}{2}\frac{1}{3}$ Engliſh ‘ legal buſhels; and the *Modius* was the ‘ third part of the *Amphora.*’

PROP. I. PROB.

To find the ſolid content of a given Priſm.

BY the 2d Prop. of the 2d Part of this, let the area of the baſe of the priſm be meaſured, and be multiplied by the height of the priſm, the product will give the ſolid content of the priſm.

PROP. II. PROB.

To find the ſolid content of a given Pyramid.

THE area of the baſe being found, (by the 3d Prop. of the 2d Part), let it be multiplied by the third part of the height of the pyramid, or the third part of the baſe by the height, the product will give the ſolid content, by 7th, 12. Eucl.

COROLLARY.

If the ſolid content of a *fruſtum* of a pyramid is required, firſt let the ſolid content of the entire pyramid be found; from which ſubtract the ſolid content of the part that is wanting, and the ſolid content of the broken pyramid will remain.

PROP. III. PROB.

To find the content of a given Cylinder.

THE area of the baſe being found, (by Prop. 6. of the ſecond part), if it be a circle, and by Prop. 8. if it be an ellipſe, (for in both caſes it is a cylinder, multiply it by the height of the cylinder, and the ſolid content of the cylinder will be produced.

COROLLARY FIG. 1.

And in this manner may be meaſured the ſolid contents of veſſels and caſks not much different from a cylinder, as ABCD. If towards the middle EF it be ſomewhat groſſer, the area of the circle of the baſe being found (by 6th Prop. of the 2d Part) and added to the area of the middle circle EF, and the half of their ſum (that is, an arithmetical mean between the area of the baſe, and the area of the middle circle) taken for the baſe of the veſſel, and multiplied into its height, the ſolid content of the given veſſel will be produced.

Note, That the length of the veſſel, as well as the diameter of the baſe, and of the circle EF, ought to be taken within the ſtaves; for it is the content within the ſtaves that is ſought.

PROP. IV. PROB.

To find the ſolid content of a given Cone.

LET the area of the baſe (found by Prop. 6. 2d Part) be multiplied into $\frac{1}{3}$ of the height, the product will give the ſolid content of the cone; for, by 10th 12. Eucl. a cone is the third part of a cylinder that has the ſame baſe and height.

PROP. V.

PROB. FIG. 2. and 3.

To find the ſolid content of a fruſtum *of a cone cut by a plane parallel to the plane of the baſe.*

FIRST, let the height of the entire cone be found, and thence (by the preceding Prop.) its ſolid content; from which ſubtract the ſolid content of the cone cut off at the top, there will remain the ſolid content of the *fruſtum* of the cone.

How the content of the entire cone may be found, appears thus: Let ABCD be the *fruſtum* of the cone (either right or ſcalenous as the figures 2. and 3.): Let the cone ECD be ſuppoſed to be compleat: Let AG be drawn parallel to DE, and AH and EF be perpendicular on CD; it will be (by 2d 6. Eucl.) as CG : CA :: CD : CE; but (by the 4th Prop. of the ſame book) as CA : AH :: CE : EF;

CE : EF; conſequently (by 22d 5. Eucl.) as CG : AH :: CD : EF; that is, as the exceſs of the diameter of the leſſer baſe is to the height of the *fruſtum*, ſo is the diameter of the greater baſe to the height of the entire cone.

COROLLARY. FIG. 4.

Some caſks whoſe ſtaves are remarkably bended about the middle, and ſtraight towards the ends, may be taken for two portions of cones, without any conſiderable error. Thus ABEF is a *fruſtum* of a right cone, to whoſe baſe EF, on the other ſide, there is another ſimilar *fruſtum* of a cone joined EDCF. The vertices of theſe cones, if they be ſuppoſed to be compleated, will be found at G and H. Whence, by the preceding Prop. the ſolid content of ſuch veſſels may be found.

PROP.

11.0.15.17.1

PROP. VI.

PROB. FIG. 5.

A CYLINDER circumſcribed about a ſphere, that is, having its baſe equal to a great circle of the ſphere, and its height equal to the diameter of the ſphere, is to the ſphere as 3 to 2.

Let ABEC be the quadrant of a circle, and ABDC the circumſcribed ſquare; and likewiſe the triangle ADC; by the revolution of the figure about the right line AC, as axis, a hemiſphere will be generated by the quadrant, a cylinder of the ſame baſe and height by the ſquare, and a cone by the triangle. Let theſe three be cut any how by the plane HF, parallel to the baſe AB, the ſection in the cylinder will be a circle whoſe radius is FH, in the hemiſphere a circle of the radius EF, and in the cone a circle of the radius GF.

By the 47th 1. Eucl. EAq, or HFq=EFq and FAq taken together, (but AFq=FGq, becauſe AC=CD); therefore the circle of the radius HF is equal to a circle of the radius EF, together with a circle of the radius GF; and, ſince this is true every where, all the circles together deſcribed by the reſpective *radii* HF (that is the cylinder) are equal to all the circles deſcribed by the reſpective *radii* EF and FG (that is, to the hemiſphere and the cone taken together); but, by 10th 12. Eucl. the cone generated by the triangle DAC is one third part of the cylinder generated by the ſquare BC. Whence it follows, that the hemiſphere generated by the rotation of the quadrant ABEC is equal to the remaining two third parts of the cylinder, and that the whole ſphere is $\frac{2}{3}$ of the double cylinder circumſcribed about it.

This is that celebrated 39th Prop. 1. book of Archimedes of the ſphere and cylinder, in which he determines the proportion of the cylinder to the ſphere inſcribed to be that of 3 to 2.

COROL-

COROLLARY 1.

Hence it follows, that the ſphere is equal to a cone whoſe height is equal to the ſemidiameter of the ſphere, having for its baſe a circle equal to the ſuperficies of the ſphere, or to four great circles of the ſphere, or to a circle whoſe radius is equal to the diameter of the ſphere, by 14th Prop. 2d part of this. And indeed a ſphere differs very little from the ſum of an infinite number of cones that have their baſes in the ſurface of the ſphere, and their common vertex in the center of the ſphere; ſo that the ſuperficies of the ſphere, (of whoſe dimenſion ſee 14th Prop. 2d part of this) multiplied into the third part of the ſemidiameter, gives the ſolid content of the ſphere.

PROP. VII.

PROB. FIG. 6.

To find the Solid Content of a Sector of the Sphere.

A Spherical Sector ABC (as appears by the Corol. of the preceding Prop.) is very little different from an infinite number of cones, having their baſes in the ſuperficies of the ſphere BEC, and their common vertex in the center. Wherefore the ſpherical ſuperficies BEC, being found (by 15. Prop. 2d part), and multiplied into the third part of AB the radius of the ſphere, the product will give the ſolid content of the ſector ABC.

COROLLARY.

It is evident how to find the ſolidity of a ſpherical ſegment leſs than a hemiſphere, by ſubtracting the cone ABC from the ſector already found. But, if the ſpherical ſegment be greater than a hemiſphere, the cone correſponding muſt be added to the ſector, to make the ſegment.

P R O P. VIII.

PROB. FIG. 7.

To find the ſolidity of the ſpheroid, and of its ſegments cut by planes perpendicular to the axis.

IN the 2d Prop. of this part it is ſhown, that every where EH : EG :: CF : CD; but circles are as the ſquares deſcribed upon their rays; that is, the circle of the radius EH is to the circle of the radius EG, as CF*q* to CD*q*. And ſince it is ſo every where, all the circles deſcribed with the reſpective rays EH, (that is, the ſpheroid made by the rotation of the ſemi-ellipſis AFB around the axis AB) will be to all the circles deſcribed by the reſpective *radii* EG, (that is, the ſphere deſcribed by the rotation of the ſemicircle ADB on the axis AB) as FC^q to CD^q; that is, as the ſpheroid to the ſphere on the ſame axis, ſo is the ſquare of the other axis of the generating ellipſe to the ſquare of the axis of the ſphere.

And this holds, whether the ſpheroid be found by a revolution around the greater or leſſer axis.

COROLLARY 1.

Hence it appears, that the half of the ſpheroid, formed by the rotation of the ſpace AHFC around the axis AC, is double of the cone generated by the triangle AFC about the ſame axis; which is the 32d Prop. of Archimedes, of conoids and ſpheroids.

COROLLARY 2.

Hence likewiſe is evident the meaſure of ſegments of the ſpheroid cut by planes perpendicular to the axis. For the ſegment of the ſpheroid made by the rotation of the ſpace ANHE, round the axis AE, is to the ſegment of the ſphere having the ſame axis AC, and made by the rotation of the ſegment of the circle AMGE, as CF*q* to CD*q*.

But, if the meaſure of this ſolid be wanted with leſs labour by the 34th Prop. of Archimedes, of conoids and ſpheroids, it will be as BE to AC+EB, ſo is the cone generated by the rotation of the triangle AHE round the axis AE, to the ſegment of the ſphere made by the rotation of the ſpace ANHE round the ſame axis AE; which could eaſily be demonſtrated (was this a proper place for it) by the method of indiviſibles.

COROLLARY 3.

Hence it is eaſy to find the ſolid content of the ſegment of a ſphere or ſpheroid intercepted between two parallel planes, perpendicular to the axis. This agrees as well to the oblate as to the oblong ſpheroid; as is obvious.

COROLLARY 4. FIG. 8.

If a caſk is to be valued as the middle piece of an oblong ſpheroid, cut by the two planes DC and FG, at right angles to the axis: Firſt, Let the ſolid content of the half ſpheroid ABCED be meaſured by the preceding Prop. from which let the ſolidity of the ſegment DEC be ſubtracted, and there will remain the ſegment ABCD; and this doubled will give the capacity of the caſk required

The following method is generally made uſe of for finding the ſolid content of ſuch veſſels. The double area of the greateſt circle, that is, of that which is deſcribed by the diameter AB at the middle of the caſk, is added to the area of the circle at the end, that is, of the circle DC or FG (for they are uſually equal), and the third part of this ſum is taken for a mean baſe of the caſk; which therefore multiplied into the length of the caſk OP, gives the content of the veſſel required.

Sometimes veſſels have other figures different from thoſe we have mentioned; the eaſy methods of meaſuring which may be learned from thoſe who practiſe this art. What hath already been delivered is ſufficient for our purpoſe.

PROP. IX.

PROB. FIG. 9. and 10.

To find how much is contained in a veſſel that is in part empty, whoſe axis is parallel to the horizon.

LET AGBH be the great circle in the middle of the caſk, whoſe ſegment GBH is filled with liquor, the ſegment GAH being empty; the ſegment GBH is known, if the depth EB be known, and EH

a

a mean proportional between the ſegments of the diameter AB and EB; which are found by a rod or ruler put into the veſſel at the orifice. Let the baſis of the caſk, at a medium, be found, which ſuppoſe to be the circle CKDL; and let the ſegment KCL be ſimilar to the ſegment GAH (which is either found by the rule of three, becauſe, as the circle AGBH is to the circle CKDL, ſo is the ſegment GAH to the ſegment KCL; or is found from the tables of ſegments made by authors); and the product of this ſegment multiplied by the length of the caſk will give the liquid content remaining in the caſk.

PROP. X. PROB.

To find the ſolid content of a regular and ordinate body.

A Tetraedron being a pyramid, the ſolid content is found by the 2d Prop. of this part. The Hexaedron, or cube being a kind of priſm, it is meaſured by the 1ſt Prop. of this part. An Octaedron conſiſts of two pyramids of the ſame ſquare baſe and of equal heights; conſequently its meaſure is found from the 2d Prop. of this part. A Dodecaedron conſiſts of twelve pyramids having equal equilateral and equiangular pentagonal baſes; and ſo one of theſe being meaſured (by 2d Prop. of this) and multiplied by 12, the product will be equal to the ſolid content of the Dodecaedron. The Icoſiaedron conſiſts of 20 equal pyramids having triangular baſes; the ſolid content of one of which being found (by the 2d Prop. of this) and multiplied by 20, gives the whole ſolid. The baſes and heights of theſe pyramids, if you want to proceed more exactly, may be found by Trigonometry.

PROP. XI. PROB.

To find the ſolid content of a body, however irregular.

LET the given body be immerſed into a veſſel of water, having the figure of a parallelopipedon or priſm, and let it be noted how much the water is raiſed upon the immerſion of the body. For it is plain that the ſpace which the water fills, after the immerſion of the body, exceeds the ſpace filled before its immerſion, by a ſpace equal to the ſolid content of the body, however irregular. But, when this exceſs is of the figure of a parallelopipedon or priſm, it is eaſily meaſured by the firſt Prop. of this part, to wit, by multiplying the area of the baſe, or mouth of the veſſel, into the difference of the elevations of the water before and after immerſion. Whence is found the ſolid content of the body given. *Q. E. I.*

In the ſame way the ſolid content of a part of a body may be found, by immerſing that part only in water.

There is no neceſſity to inſiſt here on diminiſhing or enlarging ſolid bodies in a given proportion. It will be eaſy to deduce theſe things from the 11th and 12th books of Euclid.

‘ The following rules are ſubjoined for ‘ the ready computation of the contents of ‘ veſſels, and of any ſolids, in the meaſures ‘ in uſe in Great Britain.

‘ I. To find the content of a cylindric ‘ veſſel in Engliſh wine-gallons, the diameter of the baſe and altitude of the veſſel ‘ being given in inches and decimals of an ‘ inch.

‘ Square the number of inches in the diameter of the veſſel; multiply this ſquare ‘ by the number of inches in the height: ‘ Then multiply the product by the decimal ‘ fraction .0034; and this laſt product ſhall ‘ give the content in wine-gallons and decimals of ſuch a gallon. To expreſs the

‘ rule arithmetically. Let D repreſent the
‘ number of inches and decimals of an inch
‘ in the diameter of the veſſel, and H the
‘ inches and decimals of an inch in the
‘ height of the veſſel; then the content in
‘ wine-gallons ſhall be $DDH \times \frac{34}{10000}$, or
‘ $DDH \times .0034$. *Ex.* Let the diameter
‘ $D = 51.2$ inches, the height $H = 62.3$ inches,
‘ then the content ſhall be $51.2 \times 51.2 \times 62.3$
‘ $\times .0034 = 555.27,342$ wine-gallons. This
‘ rule follows from Prop. 7. of the ſecond
‘ part, and Prop. 3. of the third part; for, by
‘ the former, the area of the baſe of the veſ-
‘ ſels is in ſquare inches $DD \times 7854$; and, by
‘ the latter, the content of the veſſel in ſolid
‘ inches is $DDH \times .7854$; which divided by
‘ 231 (the number of cubical inches in a
‘ wine-gallon) gives $DDH \times .0034$, the con-
‘ tent in wine-gallons. But, though the
‘ charges in the exciſe are made (by ſtatute)
‘ on the ſuppoſition that the wine-gallon
‘ contains 231 cubical inches; yet it is ſaid,
‘ that, in ſale, 224 cubical inches, the con-
‘ tent of the ſtandard meaſured in Guildhall
‘ (as

‘ (as was mentioned above) are allowed to ‘ be a wine-gallon.

‘ II. Suppoſing the Engliſh ale-gallon to ‘ contain 282 cubical inches, the content of ‘ a cylindric veſſel is computed, in ſuch gallons, by multiplying the ſquare of the diameter of a veſſel by its height, as formerly, and their product by the decimal fraction .0,027,851. That is, the ſolid content ‘ in ale-gallons is DDH × .0,027.851.

‘ III. Suppoſing the Scots pint to contain ‘ about 103.4 cubical inches, (which is the ‘ meaſure given by the ſtandards at Edinburgh, according to experiments mentioned above), the content of a cylindric veſſel is computed in Scots pints, by multiplying the ſquare of the diameter of the ‘ veſſel by its height, and the product of ‘ theſe by the decimal fraction .0076. Or ‘ the content of ſuch a veſſel in Scots pints ‘ is DDH × .0076.

‘ IV. Suppoſing the Wincheſter buſhel ‘ to contain 2187 cubical inches, the content of a cylindric veſſel is computed in

‘ thoſe buſhels by multiplying the ſquare of ‘ the diameter of the veſſel by the height, ‘ and the product by the decimal fraction ‘ .0,003,606. But the ſtandard buſhel having been meaſured by Mr Everard and ‘ others in 1696, it was found to contain ‘ only 2145.6 ſolid inches; and therefore it ‘ was enacted, in the act for laying a duty ‘ upon malt, *That every round buſhel, with* ‘ *a plain and even bottom, being* 18½ *inches* ‘ *diameter throughout, and* 8 *inches deep,* ‘ *ſhould be eſteemed a legal* Wincheſter *buſhel.* According to this act (ratified in the ‘ firſt year of Queen Anne) the legal Wincheſter buſhel contains only 2150.42 ſolid inches. And the content of a cylindric veſſel is computed in ſuch buſhels, by ‘ multiplying the ſquare of the diameter by ‘ the height, and their product by the decimal fraction .0,003,625. Or the content of the veſſel in thoſe buſhels is ‘ DDH × .0,003,625.

‘ V. Suppoſing the Scots wheat firlot to ‘ contain 21½ Scots pints, (as is appointed

‘ by the ſtatute 1618), and the pint to be ‘ conform to the Edinburgh ſtandards above ‘ mentioned, the content of a cylindric veſſel ‘ in ſuch firlots is computed by multiplying ‘ the ſquare of the diameter by the height, ‘ and their product by the decimal fraction ‘ .00,358. This firlot, in 1426, is appointed to contain 17 pints; in 1457, it was appointed to contain 18 pints; in 1587, it is ‘ 19¼ pints; in 1628, it is 21¼ pints: And ‘ though this laſt ſtatute appears to have ‘ been founded on wrong computations in ‘ ſeveral reſpects; yet this part of the act ‘ that relates to the number of pints in the ‘ firlot ſeems to be the leaſt exceptionable; ‘ and therefore we ſuppoſe the firlot to contain 21¼ pints of the Edinburgh ſtandard, ‘ or about 2197 cubical inches; which a little exceeds the Wincheſter buſhel, from ‘ which it may have been originally copied.

‘ VI. Suppoſing the bear-firlot to contain ‘ 31 Scots pints, (according to the ſtatute ‘ 1618), and the pint conform to the Edinburgh ſtandards, the content of a cylindric veſſel in ſuch firlots is found by multiplying the ſquare of the diameter by the height, and this product by .000,245.

‘ When the ſection of the veſſel is not a circle, but an ellipſis, the product of the greateſt diameter by the leaſt, is to be ſubſtituted in thoſe rules for the ſquare of the diameter.

‘ VII. To compute the content of a veſſel that may be conſidered as a *fruſtum* of a cone in any of thoſe meaſures.

‘ Let A repreſent the number of inches in the diameter of the greater baſe, B the number of inches in the diameter of the leſſer baſe. Compute the ſquare of A, the product of A multiplied by B, and the ſquare of B, and collect theſe into a ſum. Then find the third part of this ſum, and ſubſtitute it in the preceding rules in the place of the ſquare of the diameter; and proceed in all other reſpects as before. Thus, for example, the content in wine-gallons is $\overline{AA \times AB \times BB} \times \frac{1}{3} \times H \times .0034$.

‘ Or, to the ſquare of half the ſum of the ‘ diameters A and B, add one third part of ‘ the ſquare of half their difference, and ‘ ſubſtitute this ſum in the preceding rules ‘ for the ſquare of the diameter of the ‘ veſſel; for the ſquare of $\frac{1}{2}$A × $\frac{1}{2}$B added ‘ to $\frac{1}{3}$ of the ſquare of $\frac{1}{2}$A—$\frac{1}{2}$B, gives $\frac{1}{3}$ ‘ AA × $\frac{1}{2}$ AB × $\frac{1}{3}$ BB.

‘ VIII. When a veſſel is a *fruſtum* of a ‘ parabolic conoid, meaſure the diameter of ‘ the ſection at the middle of the height of ‘ the *fruſtum*; and the content will be ‘ preciſely the ſame as of a cylinder of this ‘ diameter, of the ſame height with the ‘ veſſel.

‘ IX. When a veſſel is a *fruſtum* of a ‘ ſphere, if you meaſure the diameter of the ‘ ſection at the middle of the height of the ‘ *fruſtum*, then compute the content of a ‘ cylinder of this diameter of the ſame ‘ height with the veſſel, and from this ſubſtract $\frac{1}{3}$ of the content of a cylinder of the ‘ ſame height, on a baſe whoſe diameter is ‘ equal to its height; the remainder will

‘ give the content of the veſſel. That is, if ‘ D repreſent the diameter of the middle ‘ ſection, and H the height of the *fruſtum*, ‘ you are to ſubſtitute $DD - \frac{1}{3} HH$ for the ‘ ſquare of the diameter of the cylindric ‘ veſſel in the firſt ſix rules.

‘ X. When the veſſel is a *fruſtum* of a ‘ ſpheroid, if the baſes are equal, the con- ‘ tent is readily found by the rule in p. 104. ‘ In other caſes, let the axis of the ſolid be ‘ to the conjugate axis, as *n* to 1; let D be ‘ the diameter of the middle ſection of the ‘ *fruſtum*, H the height or length of the ‘ *fruſtum*; and ſubſtitute in the firſt ſix ‘ rules $DD - \frac{HH}{3nn}$ for the ſquare of the ſquare ‘ of the diameter of the veſſel.

‘ XI. When the veſſel is an hyperbolic ‘ conoid, let the axis of the ſolid be to the ‘ conjugate axis, as *n* to 1, D the diameter of ‘ the ſection at the middle of the *fruſtum*, H ‘ the height or the length: Compute DD ‘ $\times \frac{1}{3nn} \times HH$, and ſubſtitute this ſum for ‘ the ſquare of the diameter of the cylindric ‘ veſſel in the firſt ſix rules.

'XII. In general, it is uſual to meaſure 'any round veſſel, by diſtinguiſhing it into 'ſeveral *fruſtums*, and taking the diameter 'of the ſection at the middle of each *fruſtum*; 'thence to compute the content of each, as 'if it was a cylinder of that mean diameter; 'and to give their ſum as the content of 'the veſſel. From the total content, com'puted in this manner, they ſubtract ſuc'ceſſively the numbers which expreſs the 'circular areas that correſpond to thoſe 'mean diameters, each as often as there are 'inches in the altitude of the *fruſtum* to 'which it belongs, beginning with the up'permoſt; and in this manner calculate a 'table for the veſſel, by which it readily ap'pears how much liquor is at any time con'tained in it, by taking either the dry or 'wet inches; having regard to the inclina'tion or drip of the veſſel, when it has any.

'This method of computing the content 'of a *fruſtum* from the diameter of the ſec'tion at the middle of its height, is exact 'in that caſe only when it is a portion of a ‘parabolic conoid; but in ſuch veſſels as are in common uſe, the error is not conſiderable. When the veſſel is a portion of a cone or hyperbolic conoid, the content, by this method, is found leſs than the truth; but, when it is a portion of a ſphere or ſpheroid, the content computed in this manner exceeds the truth. The difference or error is always the ſame, in the different parts of the ſame or of ſimilar veſſels, when the altitude of the *fruſtum* is given. And when the altitudes are different, the error is in the triplicate *ratio* of the altitude. If exactneſs be required, the error in meaſuring the *fruſtum* of a conical veſſel, in this manner, is $\frac{1}{4}$ of the content of a cone ſimilar to the veſſel, of an altitude equal to the height of the *fruſtum*. In a ſphere, it is $\frac{1}{3}$ of a cylinder, of a diameter and height equal to the *fruſtum*. In the ſpheroid and hyperbolic conoid, it is the ſame as in a cone generated by the right angled triangle, contained by the two ſemiaxes of the figure, revolving about 'that ſide which is the ſemiaxis of the '*fruſtum*. Theſe are demonſtrated in a trea'tiſe of fluxions by Mr Colin M'Laurin, p. '22. and 715. where thoſe theorems are 'bounded by planes oblique to the axis in 'all the ſolids that are generated by any 'conic ſection revolving about either axis.

'In the uſual method of computing a 'table for a veſſel, by ſubducting from the 'whole content the number that expreſſes 'the uppermoſt area, as often as there are 'inches in the uppermoſt *fruſtum*, and after'wards the numbers for the other areas 'ſucceſſively; it is obvious that the contents 'aſſigned by the table, when a few of the 'uppermoſt inches are dry, are ſtated a lit'tle too high, if the veſſel ſtands on its leſ'ſer baſe, but too low when it ſtands on 'its greater baſe; becauſe, when one inch 'is dry, for example, it is not the area at 'the middle of the uppermoſt *fruſtum*, but 'rather the area at the middle of the upper'moſt inch, that ought to be ſubducted 'from the total content, in order to find 'the content in this caſe.

‘ XIII. To meaſure round timber, let the mean circumference be found in feet and decimals of a foot; ſquare it; multiply this ſquare by the decimal .079,577, and the product by the length; Ex. Let the mean circumference of a tree be 10.3 feet, and the length 24 feet. Then 10.3 × 10.3 × 079,577 × 24 = 202.615, is the number of cubical feet in the tree. The foundation of this rule is, that, when the circumference of a circle is 1, the area is .0,795,774,715, and that the areas of circles are as the ſquares of their circumferences.

‘ But the common way uſed by artificers for meaſuring round timber, differs much from this rule. They call one fourth part of the circumference the *girt*, which is by them reckoned the ſide of a ſquare, whoſe area is equal to the area of the ſection of the tree; therefore they ſquare the *girt*, and then multiply by the length of the tree. According to their method, the tree of the laſt example would be computed at 159.13 cubical feet only.

'How ſquare timber is meaſured will ' be eaſily underſtood from the preceding ' Propoſitions. Fifty ſolid feet of hewn ' timber, and forty of rough timber, make ' a load.

' XIV. To find the burden of a ſhip, or ' the number of tons it will carry, the following rule is commonly given. Multiply the length of the keel taken within ' board, by the breadth of the ſhip within ' board, taken from the mid-ſhip beam from ' plank to plank, and the product by the ' depth of the hold, taken from the plank ' below the keelſon to the under part of the ' upper deck plank, and divide the product ' by 94, the quotient is the content of the ' tonnage required. This rule, however, ' cannot be accurate; nor can one rule be ' ſuppoſed to ſerve for the meaſuring exactly the burden of ſhips of all ſorts. Of this ' the reader will find more in the Memoirs ' of the Royal Academy of Sciences at Paris ' in the year 1721.

'Our author having ſaid nothing of weights, it may be of uſe to add briefly, that the Engliſh Troy-pound contains 12 ounces, the ounce 20 penny weight, and the penny weight 24 grains; that the Averdupois pound contains 16 ounces, the ounce 16 drams, and that 112 pounds is uſually called the hundred weight. It is commonly ſuppoſed that 14 pounds Averdupois are equal to 17 pounds Troy. According to Mr Everard's experiments, one pound Averdupois is equal to 14 ounces 11 penny-weight and 16 grains Troy, that is, to 7000 grains; and an Averdupois ounce is 437$\frac{1}{2}$ grains. The Scots Troy-pound (which, by the ſtatute 1718, was to be the ſame with the French) is commonly ſuppoſed equal to 15$\frac{3}{4}$ ounces Engliſh Troy, or 7560 grains. By a mean of ſtandards kept by the Dean of Guild of Edinburgh, it is 7599$\frac{3}{32}$ or 7600 grains. They who have meaſured the weights which were ſent from London, after the union of the kingdoms, to be the ſtandards 'by which the weights in Scotland ſhould 'be made, have found the Engliſh Averdupois pound (from a medium of the ſeveral 'weights) to weigh 7000 grains, the ſame 'as Mr Everard; according to which, the 'Scots, Paris, or Amſterdam pound, will 'be to the pound Averdupois as 38 to 35. 'The Scots Troy-ſtone contains 16 pounds, 'the pound two marks or 16 ounces, an 'ounce 16 drops, a drop 36 grains. Twenty Scots ounces make a Tron pound; 'but, becauſe it is uſual to allow one to 'the ſcore, the Tron pound is commonly '21 ounces. Sir John Skene, however, 'makes the Tron ſtone to contain only '19½ pounds.'

FINIS.

9